Rudin, page 239/21b: We'll need the derivative:

$$f'(x,y) = (6x^2 - 6x, 6y^2 + 6y) = 6(x(x-1), y(y+1))$$

For functions $f :^2 \to$, the implicit function theorem allows us to solve $f(x,y) = 0$ locally for x in terms of y provided the partial derivative of f with respect to x does not vanish—i.e. provided $x \neq 0, 1$. Let us find out where on the zero level set of f this happens:

$$\begin{aligned} 0 &= f(0,y) = 2y^3 + 3y^2 \Rightarrow y = 0, -3/2. \\ 0 &= f(1,y) = -1 + 2y^3 + 3y^2 \Rightarrow y = -1, 1/2 \end{aligned}$$

So by the implicit function theorem, we can solve locally for x in terms of y except near the points $(0,0), (0,-3/2), (1,-1), (1,1/2)$.

Now if the goal is to solve for y in terms of x, then the implicit function theorem allows us to do so at any point on the zero level set where the partial derivative of f with respect to y vanishes. The problem points occur when $y = 0$ or $y = -1$, and $2x^3 - 3x^2 + 2y^3 + 3y^2$. More specifically we can solve for y in terms of x except at $(0,3/2), (0,0), (1,-1), (-1/2,-1)$.