

**Rudin, page 239/23:** The linear approximation of  $f(x, y_1, y_2)$  about the point  $(0, 1, -1)$  is

$$L(x, y_1, y_2) = f(0, 1, -1) + f'(0, 1, -1) \begin{pmatrix} x \\ y_1 - 1 \\ y_2 + 1 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix} + \begin{pmatrix} 1 & 0 & 1 \end{pmatrix} \begin{pmatrix} x \\ y_1 - 1 \\ y_2 + 1 \end{pmatrix} = x + y_2 - 1.$$

One way to restate the implicit function theorem is to say that you can locally solve  $f = 0$  for some of the variables in terms of the others if you can do the same for the linear approximation. So that's we do here—we want to solve  $f(x, y_1, y_2) = 0$  for  $x$  in terms of  $y_1$  and  $y_2$ , so we replace  $f$  by  $L$  and get

$$x + y_2 - 1 = 0 \Rightarrow x = 1 - y_2,$$

where the right side is the linear approximation of the implicit function  $x = g(y_1, y_2)$ . In particular,

$$\frac{\partial g}{\partial y_1}(1, -1) = 0, \quad \frac{\partial g}{\partial y_2}(1, -1) = -1.$$