

Rudin, page 239/6: Since f is a rational function whose denominator vanishes only at the origin, it is clear that the partial derivatives of f exist and are continuous everywhere except $(x, y) = (0, 0)$. Now at the origin, we have

$$D_1f(0, 0) = \lim_{h \rightarrow 0} \frac{f(h, 0) - f(0, 0)}{h} = \lim_{h \rightarrow 0} \frac{h \cdot 0 - 0}{h^2 + 0} = \lim_{h \rightarrow 0} \frac{0}{h^2} = 0.$$

A similar computation shows that $D_2f(0, 0) = 0$.

However, if we consider the sequence of points $p_n = (1/n, 1/n)$, then

$$\lim_{n \rightarrow \infty} f(p_n) = \lim_{n \rightarrow \infty} \frac{1/n^2}{2/n^2} = \frac{1}{2} \neq 0 = f(0, 0) = f(\lim_{n \rightarrow \infty} p_n).$$

So f is not continuous at $(0, 0)$.