Rudin, page 239/7: Let $p = (x_1, \dots, x_n), q = (y_1, \dots, y_n) \in E$ be any two points. For $k = 1, \dots, n$ let $p_k = (x_1, \dots, x_k, y_{k+1}, \dots, y_n)$ (in particular $p_n = p$), and set $p_0 := 0$. Then for $1 \leq k \leq n$, the points p_k and p_{k-1} differ only in the kth coordinate. Hence, by the one-variable mean value theorem, there exists c_k between x_k and y_k such that

$$|f(p_k) - f(p_{k-1})| = |D_k f(c_k)(x_k - y_k)| \leq C x_k - y_k,$$

where $C > 0$ is an upper bound for $D_1 f, \dots D_n f$ on E. Hence

$$|f(p) - f(q)| \leq \sum_{k=1}^{n} |f(p_k) - f(p_{k-1})| \leq C \sum_{k=1}^{n} |x_k - y_k| \leq Cnp - q.$$

The main thing is that the constant Cn has nothing to do with p or q.

Now let $p \in E$ be any point and $\{p_n\} \subset E$ be any sequence convergin to p. Then

$$0 \leq \lim_{n \to \infty} f(p_n) - f(p) \leq C \lim_{n \to \infty} p_n - p = 0.$$

In other words $\lim_{n \to \infty} f(p_n) = f(p)$, which shows that f is continuous at p. Since p was arbitrary, we conclude that f is continuous on E.