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Rudin, page 239/8: Since f is differentiable at x, all partial derivatives $Df_1, \dots Df_n$ exist at x and $Df(x) = (Df_1(x), \dots, Df_n(x))$. And if f has a local maximum at the point $x = (x_1, \dots, x_n)$, then for any $1 \leq k \leq n$, the one-variable function

$$g_k(t) := f(x_1, \dots, x_{k-1}, t, x_{k+1}, \dots, x_n)$$

has a local maximum at $t = x_k$. In particular $g_k'(x_k) = 0$. But $g_k'(x_k)$ is just $D_k f(x)$. Hence $Df(x) = (0, \dots, 0)$.