Solutions to Homework 8

[11pt]article graphicx amssymb epstopdf

Beals, page 22 #9: No, it's not always true. Consider for example the function $f :\to$ which is zero everywhere except on intervals of the form $[n - 1/n^2, n + 1/n^2]$, $n \in$; and on such an interval the graph of f is the pair of lines joining the points $(n - 1/n^2, 0)$ and $(n + 1/n^2, 0)$ to the point $(n, 1)$. Then f is continuous and $f(n) = 1$ for every $n \in$, so $\lim_{x \to \infty} f(x) \neq 0$ (in fact, the limit doesn't exist). However,

$$\int f = \sum_{n=1}^{\infty} \frac{1}{n^2} < \infty.$$