Math 366, Winter '03
Homework 5

Profs Personal Problems:

1. Consider the set $C = \{(x, y, z) \in \mathbf{R}^3 : x^2 - y^2 - z^2 = 0\}$ At which points does C fail to be an embedded submanifold? What is the dimension and codimension of C? Draw a picture of C.

2. Consider the function $h : \mathbf{R} \to \mathbf{R}^2$ given by $h(t) = (t^2, t^3)$ and the set $C = h(\mathbf{R}) \subset \mathbf{R}^2$.

- At which points does C fail to be a submanifold of $\mathbf{R}^2$?
- Describe the tangent space to C at the point $(1, -1)$.
- Draw a picture of C.

3. Consider the following set

$$M := \{(x, y, z, w) \in \mathbf{R}^4 : x^2 - y^2 - zw = 1, zy + wx = 0\}$$

- At which points does M fail to be an embedded submanifold of $\mathbf{R}^4$.
- What is the dimension and codimension of M?
- Describe (i.e. give a basis for) the tangent space of M at the point $(1, 1, 0, 0)$.
- Draw a picture of M. Full color. Stereo sound.

4. (One more time) Consider the function

$$f(x, y) = (x^3 - 2x^2y + y, y^3 - 2x^2).$$

Observe that $f(1, 1) = (0, -1)$ and verify that f has a local inverse satisfying $f^{-1}(0, -1) = (1, 1)$. Starting with a guess of $(x_0, y_0) = (1, 1)$, compute two (increasingly better) approximations of the point $(x, y) = f^{-1}(.1, -.8)$.