

Test 1 for Math 405, Introduction to Combinatorics.

Name: _____

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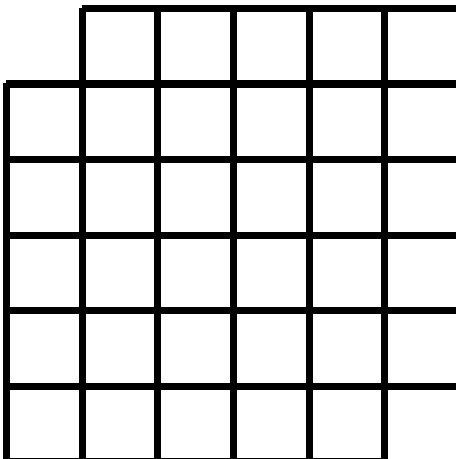
Instructions: The test will be 50 minutes in length.

1. How many sets of three numbers each can be formed from the numbers $\{1, 2, 3, \dots, 19, 20\}$ if no 2 consecutive numbers are to be in the set.

2. Calculate the (multinomial) coefficient of the monomial a^3b^2c in the expansion of $(a+b+c)^6$:

3. How many integers between 1 and 1001 are divisible by at least one of the numbers 7, 11, 13?

4. Find a cover by dominoes for the following board. Alternatively prove that no cover can exist.



5. Give a detailed proof why the formula

$$\sum_{k=0}^n \binom{n}{k}^2 = \binom{2n}{n}$$

holds for all positive integers n .

6. There are 100 people in a queue for a London cinema (and, being in England, the order of people in the queue never changes). The cinema lets the people into the show in 5 batches, each batch consisting of 15 or more persons. In how many ways can the 5 batches be chosen.

7. Find the number of integral solutions of the equation

$$x + y + z = 30$$

subject to the conditions:

$$5 \leq x \leq 15, \quad -3 \leq y \leq 12, \quad 0 \leq z \leq 20.$$

8. At a concert 100 people check their umbrella. Assume that after the concert the 100 umbrellas are returned completely at random to the 100 persons who checked the umbrellas. Find the probability that at least one person goes happily home with his own umbrella. In your answer provide both a formula and an estimate in terms of a numerical value. (Like: Formula is $\frac{\pi}{10}$ or about 0.314).