**Test 2 for Math 405, Introduction to Combinatorics.**

**Name:**______________________________ October 27, 2000

Instructions: The test will be 50 minutes in length.

1. (10 pts) Assume that
$$h_n = 5h_{n-1} - 6h_{n-2}$$
for $n \geq 3$ and that $h_1 = h_2 = 1$. Compute $h_n$ for all integers $n \geq 3$.

2. (10 pts) Assume that
$$h_{n+3} + h_{n+1} + h_n = 0$$
and that $h_1 = 0$, $h_2 = 0$ and $h_3 = 0$. Compute $h_n$ for all integers $n \geq 4$.

3. (10 pts) Let

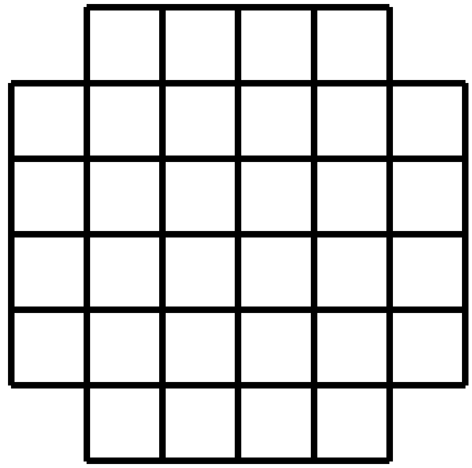
$$f(x) = \frac{1}{(x-1)(x-2)(x-3)} = \sum_{n=0}^{\infty} s_n x^n$$

be the generating function of a sequence. Find an explicit formula for $s_n$.

4. (10 pts) Solve explicitly the recurrence relation

$$\begin{aligned} h_{n+1} &= 3h_n + 3^{n+1}, \quad n \geq 1. \\ h_1 &= 2. \end{aligned}$$

5. (15 pts) What is the number of ways to place six non-attacking rooks on the following 'pruned chess-board'?



6. (15 pts) Let $h_n$ denote the number of $n$-digit sequences in which each digit is 0 or 1, no two consecutive 0's being allowed. Note that $h_1 = 2$ and $h_2 = 3$. Find an explicit formula for $h_n$. (Hint: establish a second order recurrence relation.)

7. (15 pts) Determine the number of $n$ digit numbers with all digits odd, such that 1 and 3 each occur an even number of times.

8. For each $n$ let $h_n$ denote the number of non-negative integral solutions of the equation

$$a + 3b + 5c + d + e = n$$

subject to the conditions:

$$0 \le d \le 2, \qquad 0 \le e \le 4.$$

a) (8 pts) Find the generating function of the sequence $h_n$.

b) (7 pts) When $n = 100$ compute explicitly the number of solutions.