Name: ______________________________

Math. 423 Final exam Due by Wednesday, May 10

1. Given the initial value problem $y' = \dfrac{\cos(t) - 2yt}{1+t^2}$ with $y(0) = 0$.

Use MAPLE or any other program to answer the following questions. Write the answers below and attach some documentation, such as a printout.

(a) Use the modified Euler method, first with step size $h = 0.05$ and then with $h = 0.025$, to approximate $y(1)$.

(b) Do the same using the standard Runge-Kutta method of order four.

(Continuing problem 1) The actual solution of the initial value problem is

$$y(t) = \frac{\sin(t)}{1+t^2}\,.$$

(c) Determine the errors involved in using the modified Euler method with $h = 0.05$ and $h = 0.025$. By what factor does the error improve when the step size is reduced? Is that about what one would expect? Explain.

(d) Answer the same questions for the standard Runge-Kutta method of order four.

2. Given the initial value problem $y' = -y + t, \quad y(0) = 1$.

(a) Use the modified Euler method with step size $h = 0.2$ to compute w_1.

(b) Use the Adams-Bashforth 2-step method with the w_1 of part (a) and step size $h = 0.2$ to approximate $y(0.4)$.

Continuing problem 2.

(c) Repeat parts (a) and (b) to approximate $y(0.4)$, but this time using step size $h = 0.1$; i.e., use the modified Euler method to compute w_1 and then use the Adams-Bashforth 2-step method to approximate $y(0.4)$.

Continuing problem 2.

(d) The actual solution of the initial value problem is $y(t) = t - 1 + 2e^{-t}$. Use this information to estimate the error in the approximations of $y(0.4)$ in parts (b) and (c). By what factor did the error improve using $h = 0.1$ instead of $h = 0.2$? Was that close to what you expected? Explain.

3. Given the initial value problem $y' = e^y, \quad y(0) = 0$.

(a) Use the Taylor method of order 4 with $h = 0.1$ to approximate $y(0.2)$.

(b) The actual solution of the initial value problem is $y(t) = -\ln(1-t)$. If you did part (a) correctly your answer should differ from the actual value by less than 0.0000062. Check that it does.

4. Given the initial value problem $y' = y^2 - t, \quad y(0) = 1$

(a) Use the 2-step Adams-Bashforth-Moulton predictor-corrector method to approximate $y(0.2)$ Use a step size of $h = 0.1$, and use the Taylor method of order 2 to find w_1.

(b) Use the standard 4-step Runge-Kutta method to approximate $y(0.2)$.