Math 425
February 12, 1993 Test I

1. Write in standard form: $\frac{11-2 i}{4-3 i}$
(standard from means $\mathrm{a}+\mathrm{b}$ i with $\mathrm{a} \& \mathrm{~b}$ real.)
2. Find all the cube roots of $-1+i$. Write your answers in standard form.
3. Compute $(-1+i)^{20}$. (answer in standard form)
4. Sketch the graph of $z^{-1}=\bar{z}$
5. Sketch the graph of $\operatorname{Re}((3+i) z+2-4 i)>0$.
6. Prove: if $|z|<1$, then $\operatorname{Re}\left(\frac{1+z}{1-z}\right)>0$.
7. Does the sequence $\left\{z_{n}\right\}$, where $z_{n}=i n$, converge or diverge? If it converges, what is the limit? Give reasons for either answer.
8. Discuss the convergence or divergence of $\underset{n=1}{\infty} \quad\left(\frac{i}{n}\right) n$.
9. Do the same for $\underset{n=1}{\infty} \frac{(2+i)^{n}}{(\sqrt{6})^{n}}$
10. Let $w=e^{z}$. shade the image, in the $w-$ plane, of the region $\left\{(x, y) ; \begin{array}{l}1 \leq x \leq 2 \\ 0 \leq y \leq \pi\end{array}\right\}$ under the map given by the function $e^{z}$.

11 . Find all values of $(-2)^{i}$.
12. Evaluate the line integral $\int\left(z^{4}-3 z^{3}\right) d z$, where $\gamma$ is any curve from 1 to

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1+i .
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13. Estimate $\left|\int_{\gamma} \frac{d z}{z^{4}+16}\right|$, where $\gamma$ is the circle $\gamma(t)=3 e^{i t}, 0 \leq t \leq 2 \pi$.
14. Show that $f(z)=z+\bar{z}$ is not analytic in $D$ using the Cauchy - Riemann equations.
