

March 26, 1993

I have not violated the Honor Code in any way with regard to this work.

Signature _____

1. (10 points) Determine the zeros of $f(z) = z^2 (1 - \cos z)$ and the order of each zero. Show your work.

2. (10 points) Describe the nature of the singularity of $g(z) = \frac{e^z - 1}{z}$. At which value z_0 is it located? If it is removable at z_0 , what is $g(z_0)$?

3.(4 points) Describe the poles of $f(z) = \frac{z}{z^4 - 1}$ and give the order of each one.

4. (10 points) What is the residue of $\frac{\text{Log } z}{(z - 1)^4}$ at 1 ?

5. Evaluate

a. (7 points) $\int_{\gamma} \frac{\cos z}{z} dz$ where γ is the circle $|z| = 1$.

b. (7 points) $\int_{\gamma} \frac{1}{z-2} dz$ for the same γ as in a.

c. (7 points) $\int_{\Gamma} \frac{1}{z-2} dz$ where Γ is the circle $|z|=3$.

d. (7 points) $\int_{\gamma} \frac{1}{(z-2)^7} dz$ where γ is the circle $|z-2|=5$.

6.(6 points) Let $f(z) = |z|^2$. Is $f(z)$ differentiable at $z = 0$?

7.(10 points) Evaluate $\int_{|z|=1} \frac{dz}{z(z-2)}$.

8. (6 points) The function $\frac{1}{(z-1)(z-2)}$ is analytic in a disc around the origin. What is the radius of convergence of its power series about 0 ?

9. (6 points) The integral, $\frac{1}{2\pi i} \int_{\gamma} \frac{e^z}{z^{k+1}} dz$ where γ is the circle $|z| = 1$, is one of the coefficients a_i in the power series $\sum_{n=0}^{\infty} a_n z^n$. Which one is it and what is its value? Give reasons.

10.(10 points) Find the power series expansion about 0 for the function $f(z) = \frac{3-z}{2-z}$.
What is its radius of convergence? Show your work.