1. Let $f(x) = \dfrac{1}{z^2 + 1}$. Evaluate $\int_\gamma f(z)\, dz$ where

(a) (5 points) γ is the circle centered at i, radius 1, positively oriented.

(b) (5 points) γ is the circle centered at –i, radius 1, positvely oriented.

(c) (5 points) γ is the circle centered at 0, radius $\frac{1}{2}$, positively oriented.

2. Let $f(z) = \dfrac{1}{z + 1} + \dfrac{1}{z - 2}$. Find the Laurent expansion for f(z), in powers of

z, for

(a) (5 points) $|z| < 1$

(b) (5 points) $1<|z| < 2$

c) (5 points) $|z| > 2$

3. (15 points) Let $f_n(z) = \dfrac{z}{(z-1)^n}$ where n is a positive integer. Expand $f_n(z)$ as a power series in $z - 1$, then use residue theory to evaluate $\int_\gamma f_n(z)\,dz$ for each $n > 0$.

4. (5 Points)

(a) State the argument principle in a few words.

(b) (10 points) Use it to determine the number of zeros of

$f(z) = z^4 + 3iz^2 + z - 2 + i$ in the upper half plane.

5. (a) (5 points) State Rouché's Theorem.

(b) (10 points) Use it to determine the number of zeros of $f(z) = z^6 - 5z^2 + 10$ in the annulus

$$1 < |z| < 2.$$

6. (15 points) Evaluate the integral $\displaystyle\int_{-\infty}^{\infty} \frac{x^2}{(x^2 + 1)(x^2 + 4)}\, dx.$

7. Consider the integral $\int_{\gamma} \frac{z\, e^{iz}\, dz}{z^4 + 1}\ dz$ where γ is the contour consisting of the portion of the x – axis from x = –R to x = R followed by the semicircle $z = R\, e^{i\theta}$ for θ from 0 to π.

a. (5 points) Use residue theory to evaluate the contour integral.

b. (10 points) Carefully estimate the absolute value of the integral along the semicircle so tha
can conclude that as $R \to \infty$, this part of the contour integral approaches 0.

c. (5 points) By considering the real and imaginary parts of the integral along the x– axis we v
get 2 real integrals. What are they, and what are their values?

8. (10 points) Find $\text{Res}\left(\frac{e^z}{(z+2)^3} ; -2\right)$.

9. (10 points) Find the four fourth roots of -16.

10. (10 points) Find all the values of $\log(1 + i\sqrt{3})$.

11. (10 points) Find all the values of $(1 + i)^i$.