Student's name:.....

1. (5 points) Compute

$$\frac{(2 + i)(1 - i)}{1 + i}.$$

2. (5 points) Find the number

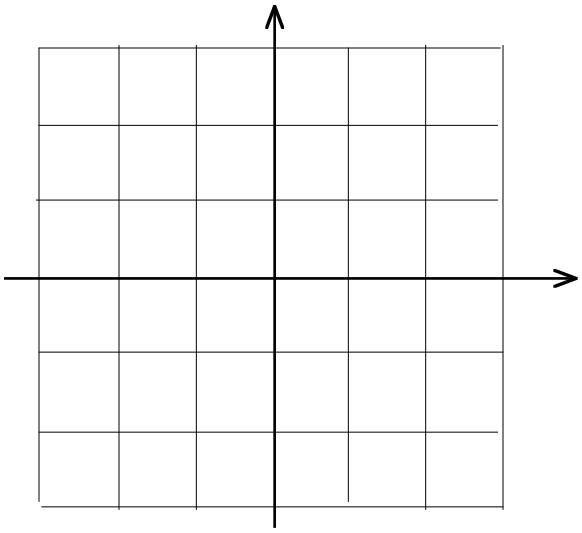
$$\left(\frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2} i \right)^{100}.$$

3. (15 points) Find all third roots of the number $8i$. Represent them in the standard form $a + bi$.

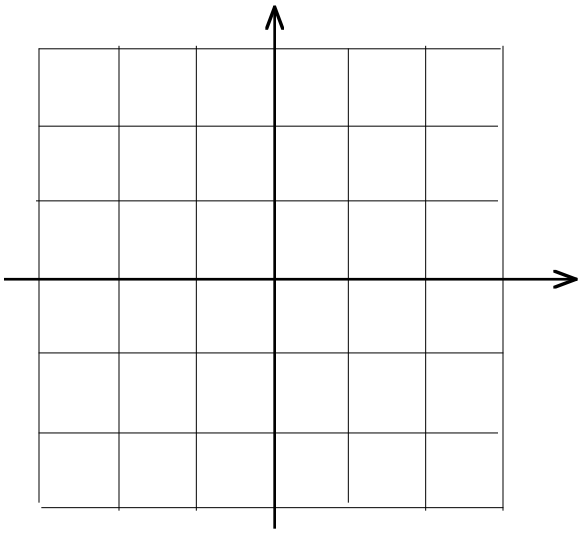
Student's name:.....

4. (10 points) Are the sets described by the following connected? Are they simply connected? Show them on the diagram.

(a) $-1 < \mathrm{Re}((1+i)z) < 1$



(b) $1 < |z - 1| < 2$



Student's name:.....

5. (20 points) Compute the following:

(a) $exp(\frac{\pi}{2}\ i)$

(b) Ln i

(c) sin i

(d) cosh i

Student's name:.....

6. (10 points) Prove the identity

$$\cos 2z = \cos^2 z - \sin^2 z$$

for every complex z.

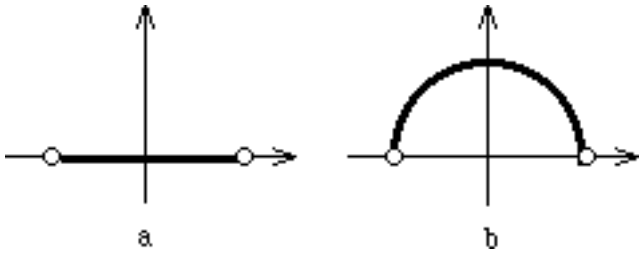
Student's name:.....

7. (20 points) Compute the line integral

$$\int_{\gamma} |z|\, z\, dz$$

(a) where γ is the the straight segment from -1 to 1,

(b) γ is the upper semicircle from -1 to 1.



Student's name:.....

8. (15 points) Use Green's theorem to prove that $\int_{\gamma} z^2\, dz = 0$, for every simple closed curve.