Student's name:.....

1. Which of the following functions is analytic?

(a) $x - 2xy + (x^2 - y^2 + y)i$, (b) $-2xy + (x^2 + y^2)i$.

Find the derivative of the analytic one.

2. Find the derivative of the function $w = \frac{az + b}{cz + d}$, where a and b are complex constants. Where does this derivative exist?

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3. Use Cauchy's theorem or Cauchy's formula to evaluate the following integrals

(a) $\int_{|z|=1} \frac{\cos z}{z-\pi}\, dz$, (b) $\int_{|z-\pi|=1} \frac{\cos z}{z-\pi}\, dz$, (c) $\int_{|z|=1} \frac{dz}{1+e^{z}}$.

4. Compute the integral $\int_{-\pi}^{\pi} \frac{dt}{2+\sin t}$.

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5. Knowing that $\sin z = \sum_{k=1}^{\infty} \frac{(-1)^k z^{2k+1}}{(2k+1)!}$, find the first three terms of the power series expansion of $\frac{\sin z}{1+z}$ about 0. Where does the latter expansion converge?

6. Find the power series expansion of the function $f(z) = \int_{-1}^{1} e^{tz}\, dz$

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7. Find all zeros of the function $f(z) = z \sin^2 z$ and determine their order.

8. Find all singularities of the function $g(z) = \exp\left(\frac{1}{z+1}\right) \frac{\sin z}{z\,(z-\frac{\pi}{2})(z-\pi)}$.

Determine their type and find the order of the poles, if any.