Math 425 **Spring semester 1994-95**

Review for Test No. 2.

1. Analytic functions. Cauchy-Riemann equations. Rules of complex differentiation. Cauchy's theorem and Cauchy's formula and their application to evaluation of integrals

<u>Examples</u>: 1. Which of the following functions are analytic and, if so, what is their derivative?

(a) $f(z) = x \sin y + i\, y \cos x$,

(b) $f(z) = (e^x + e^{-x}) \cos y + i\,(e^x + e^{-x}) \sin y$,

(c) $f(z) = (e^x + e^{-x}) \cos y + i\,(e^x - e^{-x}) \sin y$,

(d) $f(z) = x^2 + y^2 + (x^2 - y^2)\, i$.

2. Compute the derivative of the following functions. If the function is not analytic state so.

(a) $f(z) = e^{z^2+z+3}$, b) $f(z) = \sin z + \cos \bar{z}$.

(c) $f(z) = \dfrac{z^2-1}{z^2+1}$.

3. Find the following integrals:

(a) $$\int_{|z|=1} \frac{z+1}{z^4-1}\,dz\ ,$$ (b) $$\int_{|z-3|=1} \frac{z+1}{z^4-1}\,dz\ ,$$

(c) $$\int_{-1}^{1} \frac{\cos t}{2+3\sin t}\,dt$$ ~~(c)~~ $$\int_{|z-1|=1} \frac{5z^2-3z+2}{(z-1)^3}\,dz\ ,$$

2. Consequences of Cauchy's formula. Expandability of an analytic function into a power series. General Cauchy's formula. Maximum modulus principle. Morera’s theorem. Liouville's theorem. Fundamental theorem of algebra.

Example. Find the integral $\displaystyle\int_{|z-\frac{1}{2}|=1} \frac{z+1}{(z^2-1)^2}\,dz$ ..

3. Power series. Radius of convergence. Arithmetical operation on power series. Differentiation and integration of power series.

Examples: 1. What is the radius of convergence of the series $\displaystyle\sum_{n=0}^{\infty} \frac{n!}{(2n)!} z^n$.

2. Knowing that $f(z) = \displaystyle\sum_{n=0}^{\infty} \frac{n}{(n-1)!} z^n$ find the expansion of

(a) the expansion of the function $f'(z)$,

(b) the first four terms of the expansion of $\dfrac{f(z)}{1+z^2}$.

3. Find the expansion of $\sin z$ about the point $\frac{\pi}{2}$.

4. Isolated zeros of an analytic function and their order. Isolated singularities and their classification. The order of a pole. The residue of a function at a pole.

Examples: 1. Find all zeros and all singularities of the following functions. Determine the order of each zero. Determine the type of each singularity. For each removable singularity find its value. For each pole find its order.

(a) e^{z^2+z+3} ,

(b) $\frac{1}{z}\exp\left(\dfrac{z^2+z+3}{z^2-z+3}\right)$

(c) $\dfrac{2z^2 + 3z + 1}{\sin(\pi z^2)}$

(d) $\dfrac{2z^2 + 3z + 1}{\sin^2 \pi z}$