EXAM 1 MATH 431

Problem 1. In the sudy of Pythagorean numbers the problem was reduced to finding rational points (i.e., points withrational coordinates) in the unit circle $x^2 + y^2 = 1$. Use the same method to find all rational points on the curve $x^2 - y^2 = 1$.

Problem 2. Let p be a prime, show that the simultaneous equations:

$$ax + by = A \pmod{p}$$
$$cx + dy = B \pmod{p}$$

has a unique solution modulo p if $ad - bc \neq 0 \pmod{p}$.

Problem 3. Let p be a prime, solve

$$\sum_{i=1}^{p-1} i^3 = ? \pmod{p}.$$

Problem 4. Find the sum

$$\frac{1}{1 \cdot 2} + \frac{1}{2 \cdot 3} + ... + \frac{1}{(n-1) \cdot n} = ?.$$

Prove your result.

Problem 5. Three farmers divide equally the rice that they have grown. One goes to a market where an 83-pound weight is used, another goes a market where an 110-pound weight and the last goes to a market where an 135-pound weight is used. Each farmer sells as many full measure as possible and at the close of the market the first farmer has 32 pounds of rice left, the second has 70 pounds, and the third 30 pounds. Find the total amount of rice they took to the market.

Problem 6. Show that if p is prime then for any integers a and b, $(a+b)^p = a+b \pmod{p}$.