33=533 Mathematical Modelling Fall 1997
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R. Kent Nagle and Edward B. Saff, Fundamentals of Differential
quations [1993], Third Addition, Addison-Wesley Publishing Company

Richard H. Enns and George C. McGuire, Nonlinear Physics with Maple
or Scientists and Engineers [1997], Birkhauser.

ourse description includes a library of Mathematica programs developed
or the course. Each student participated in 5 projects and presented
.nal project in class.

Review of different types of mathematical models. Mathematical
ackground.

Examples of mathematical models.

a) Different population models based on linear and nonlinear
ODE's including logistic model.

b) Competition models and predator-prey models and systems of
nonlinear equations. Examples from biology and optics.

3. Modelling using first order differential equations.

a) Separable equations and exact equations. Discontinuous
solutions. Diffusion of medicine in the blood stream.

b) Equations with discontinuous coefficients. Discontinuous force in
mechanics.

c) Singular points. Nonuniqness of the solution.

d) Bernoulli equations and homogeneous equations. Ecological models.
Beam competition equations from laser optics.

e) Delay differential equations. Flow systems. Mixing problem.
Clearning of Great lakes.

f) Optimization problems. Motion of a multi-stage rocket.

g) Mathematical modelling of geometric problems.

h) Linear equations with non constant coefficients. Linear pursuit
problem.

Discrete models. Examples from economics and population dynamics.
Discrete dynamical systems. Periodic and critical points. Phase
diagrams.

a) Discrete maps including logistic map. Steady states, equilibria, fixed points. Stability of fixed points. Bifurcations.

b) Logistic map and period doubling. Chaotic systems. Pichfork diagrams. Fractals and chaotic systems with examples.

c) Discrete systems and numerical methods. Numerical chaos.

Chaotic systems. Topological approach to Chaos. Poincare-Smale theorem.

Mathematical modelling through linear ODE's of second order. Resonances in mechanics. Critically dumped motion. Beats. Nonlinear equations solvable by first order techniques. A cable suspended between 2 fixed points.

Mathematical modelling using nonlinear equations. Duffing oscillator with dissipation and friction. Hamiltonian systems. Phase space analysis. Poincare map. Perturbation: forced Duffing equation. Smale hourseshoe map. Cantor sets in R^2. Elements of symbolic dynamics. Lorenz strange attractor.

Modelling using PDE's.

a) Continuity equation in fluid dynamics. Traffic flow on a high-way. Perturbation of the model. Shock wave solutions.

b) Wave motion: Hyperbolic waves and dispersive waves. Method of characteristics. Multi valued solutions and shock waves. Soliton solutions and their application in Ocean dynamics and nonlinear optics.