University of Notre Dame

Spring of 1997

Math. 433: Topics in Applied Mathematics:
Mathematical Modeling and Industrial Applications

Instructor: Dr. Hong-Ming Yin

Course Information:

The main purpose of this course is to give an introduction to the mathematical modeling and numerical solutions of partial differential equations. The course will emphasize on how to model physical problems in practice and industry by using partial differential equations and then how to find numerical solutions to these equations. Various finite difference methods and stability analyses will be studied in this course. Some industrial applications such as ice-water melting process, thermistor device and microwave heating will be discussed in this course.

The basic requirement for taking this course is advanced calculus and differential equations. This course is suitable to senior and graduate students from mathematics, physical and engineering sciences.

Text Book:

K. W. Morton and D. F. Mayers,
Numerical Solution of Partial Differential Equations,
Cambridge University Press, 1994.

Reference Books

(1) W. F. Ames, *Numerical Methods for Partial Differential Equations*, (3rd Edition), Academic Press, Inc., New York, 1992.

(2) Leon Lapidus and George F. Pinder, *Numerical Solutions of Partial Differential Equations in Science and Engineering*, John Wiley & Sons, New York, 1982.

Daily Progress:
L1: Mathematical Modeling- an overview (Jan. 15).
L2: Derivation of the equation of the heat conduction (Jan. 17).
L3: Initial and Boundary conditions, Well-posedness (Jan. 20).
L4: Numerical scheme for One-Dimensional heat Equations(Jan.22)
L5: Truncation Error Estimates (Jan. 24).
L6: Convergence of the explicit scheme (Jan. 27).
L7: Stability- Fourier's method. (Jan 29).
L8: Implicit Difference Methods (Jan. 31).
L9: A Maximum principle and convergence for $\nu(1-\theta) < \frac{1}{2}$(Feb. 3)
L10: More General Linear Problems (Feb. 5).
L11: Finite Difference Method for nonlinear equations (Feb.7)
L12: Algorithms for finding numerical solutions of heat equations(Feb. 10)
L13: The mathematical Model of a Thermistor Device (Feb. 12).
L14: Numerical solutions of parabolic equations in two and three dimensions.(Feb. 14).
L15: Derivation of Equation of the Vibrating string (Feb.17).
L16: Characteristics and CFL condition (Feb. 19).
L17: Upwind difference scheme and Error analysis (Feb. 21).
L18: Lax-Wendroff Scheme (Feb. 24).
L19: A nonlinear conservation law. (Feb.26).
L20: Shock Wave(Feb. 28).
L21: Finite Difference Scheme for wave equations(March 3)
L22: Error estimates and convergence Theorems (March 5).
L23: Implicit Methods for wave equations (March 7).
L24: Numerical Method for Elliptic Equations (March 17).
L25: Finite Difference Method for Poisson's equations (March 19).
L26: Error Analysis by using maximum principle (March 21).
L27: Error Estimate (Part II) (March 24).
L28: Numerical Scheme for General Domains (March 26).
L29: Mathematical Model of Phase-Change: Free boundary Problems (April 2)
L30: Mathematical Model of Phase-Change (Part II)(April 4).
L31: Mathematical Model of Phase-change (Part III) (April 7).
L32: A special solution to one-phase Stefan problem (April 9).
L33: The existence of solution to one-phase,one-dimensional Stefan problem (April 11).
L34: Asymptotic behavior of the free boundary at $t = \infty$ (April 14).
L35: Long-time behavior of the free boundary (April 16)
L36: Numerical scheme for one-dimensional, one-phase Stefan problem (April 18).
L37: Some Inverse Problems (April 21).
L38: An inverse problem in half-space (April 23)
L39: Some Special solutions to some inverse problems (April 25).
L40: Mathematical Model of Microwave heating (April 28).
L41: Discussing on the results of a research paper from *Quarterly. of Applied Math.*(April 30).

Final Examination:

Part A: Take home project on finding the numerical solutions to the thermistor device.

Part B: Oral Presentation of the paper in Lecture 41.

Remark:

All lecture notes were copied and distributed to all students.