436 PDE FINAL EXAM NAME____________________

APRIL 19, 1991

1. Derive the series solution of the homogeneous damped - wave equation

$$U_{tt} + 2\,U_t - C^2 U_{xx} = 0 \quad , \qquad t \geq 0$$
$$0 \leq x \leq L$$

With the following boundary conditions

$u\,(0, t) = 0 \quad , \qquad u\,(L, t) = 0 \quad , \qquad t \geq 0$

and initial conditions be given in the form

$u\,(x,0) = 1 \quad , \qquad u_t\,(x,0) = x$

2. Denote

$$L\,(f(x)) = F\,(s) = \frac{1}{2\pi}\int_{\bullet}^{-\bullet} f(x)\, e^{-isx}\, d\,x$$

$$L^{-1}\,(F(s)) = f\,(x) = \int_{\bullet}^{-\bullet} F\,(s)\, e^{isx}\, ds\,.$$

(a) Show that

$$L\,(f(x-a)\cdot H\,(x-a)) = e^{-as}\, F\,(s)$$

and

$$L^{-1}\,(e^{-as}\, F\,(s)) = f\,(x-a)\cdot H\,(x-a)$$

where $H\,(x-a)$ is the Heaviside unit step function

$$H\,(x-a) = \begin{cases} 0 & ,\ x-a < 0 \\ 1 & ,\ x-a > 0 \end{cases}$$

(b) Use (a) to calculate Laplace transform F(s) for the following function f(x):

$$f(x) = \begin{cases} 0 & ,\ x-4 < 0 \\ x-3 & ,\ x-4 > 0 \end{cases}$$

3. Let B be the circular region

$$x^2 + y^2 \leq 1.$$

On B find in terms of the Poisson integral the unique solution of the Dirichlet problem:

$$U_{xx} + U_{yy} = 0$$

$$U(x,y) = 2 + 2 \cdot x \qquad \text{on the boundary } x^2 + y^2 = 1$$

4. Use Laplace transforms to solve the following problem

$U_t = C^2 U_{xx}$; $t \geq 0$

$0 \leq x \leq L$

$U(o,t) = 0$

$U(L,t) = 0$

$U(x,0) = 5x$

5. Use d'Alembert's formula to solve the wave equation

$U_{tt} = C^2 U_{xx}$

with the initial conditions

$y(x;0) = 0$, $y_t(x;0) = 2\cos x + 3\sin 2x$