

FINAL

Show all your work. If you find you are doing a horrendous calculation, you are making a mistake or at least making the problem unnecessarily difficult. (Some calculations are necessary.)

1. (30 points) a) Solve:

$$\begin{aligned}u_{tt} &= u_{xx}, & t > 0, 0 < x < 2, \\u(x, 0) &= 0, & 0 < x < 2, \\u_t(x, 0) &= x(2 - x), & 0 < x < 2, \\u(0, t) &= 0 = u(2, t), & t > 0.\end{aligned}$$

- b) Show that your answer solves the problem.

- c) Find $u(\frac{7}{4}, \frac{9}{4})$. (Your answer should be a number.)

2. (15 points) Find the solution $u(\rho, \phi)$ of Laplace's equation in the disk $\rho < 1$ and satisfying the boundary condition

$$u(1, \phi) = 68924 \sin 9815423\phi - 79021 \cos 4230874\phi.$$

3. (30 points) a) Solve:

$$\begin{aligned}u_t &= \nabla^2 u, & 0 < x < 2\pi, 0 < y < 2\pi, 0 < t, \\u(0, y, t) &= 0 = u(2\pi, y, t) = u(x, 0, t) = u(x, 2\pi, t), \\u(x, y, 0) &= xy(2\pi - x)(2\pi - y).\end{aligned}$$

- b) Show that your answer solves the problem.