

PROJECT, Due November 22

Your project should be a well-written explanation of the topic and should convince me that you have a good understanding of it. It should not be something copied from a book. Include references and a list of any computer programs you used. Here are some possible topics. If you want to do a topic not on this list, please discuss it with me ahead of time.

1. Learn something about numerical methods for solving pde. Implement a finite difference method on computer. If you know how to use it, use the numerical solution to graph solutions, using a powerful graphics package (e.g., the one contained in MatLab.) Discuss the accuracy of the numerical solutions.

2. Show how a computer algebra system (e.g., Mathematica, Maple) can be used to solve pde.

3. Show how a computer can be used to calculate the first mode of a “drumhead” for some non-circular shapes of drums. Make a Mathematica movie of the motion of the drum (with that mode).

4. Learn about the maximum principle and uniqueness results for the heat equation.

5. Learn about weak solutions of the wave equation (1 space variable) and learn more about how characteristics affect solutions.

6. Learn about Duhamel’s principle.

7. Learn how to solve first order quasi-linear pde.

8. Learn the theory of Sturm-Liouville problems, including a proof of completeness of the eigenfunctions under appropriate hypotheses. Learn about the asymptotic behavior of the n th eigenvalue and eigenfunction as $n \rightarrow \infty$.

9. Let

$$f(x) = \sum_{k=1}^{\infty} \frac{1}{k^2} 2 \sin(2^{k^3+1}x) \left(\sum_{j=1}^{2^{k^3}} \frac{\sin jx}{j} \right).$$

a. Show that f is continuous and 2π periodic.

b. Show that the Fourier series of f diverges at 0.

10. Learn about the problem “Can one hear the shape of a drum?”

11. Learn about the Gibbs phenomenon for general piecewise smooth functions.