Math 436

Take-home problems for final

Due at 4:15 p.m., December 11, 2000

You may consult your course notes, homework, and the textbook. You may use Maple, Matlab or Mathematica as an experimental tool on any part of the problem. If you do, please indicate how on your exam. You may not consult any other books or notes. You may not discuss the exam with anyone except me.

The in-class part of the exam will be on December 11. It will be closed book, but you may bring either a sheet of paper ($8\frac{1}{2}'' \times 11''$) with notes or two a $3'' \times 5''$ cards with notes to the exam.

You must justify your answers. If you cannot do one part, you may assume the result of that part for the remaining parts.

1. (35 points) a) Let u be a nonnegative harmonic function on $\mathbf{R}^2$ (a solution of Laplace's equation, $u_{xx} + u_{yy} = 0$). Prove that u is constant.

(**Hint**: Let $v_n(x, y) = u(nx, ny)$. Show that v_n is harmonic. Fix (x_0, y_0). What does Harnack's inequality tell you about $v_n(\frac{x_0}{n}, \frac{y_0}{n})$ as $n \to \infty$? Harnack's inequality is the inequality in #24 (a) on pp. 182-183.)

b) Prove **Liouville's Theorem**: Let u be a bounded harmonic function on $\mathbf{R}^2$. Then u is constant.

2. (30 points) A function g on $\mathbf{R}^2$ is called *radial* if there is a function ψ on $[0, \infty)$ such that $g(x, y) = \psi\left(\sqrt{x^2 + y^2}\right)$. Let f be an integrable function on $\mathbf{R}^2$. Suppose that f is radial. Show that the Fourier transform F of f is also radial.

(**Hint**: If $\mu = (\mu_1, \mu_2)$ and $\widetilde{\mu} = (\widetilde{\mu}_1, \widetilde{\mu}_2)$ with $|\mu| = |\widetilde{\mu}|$, then there an angle θ such that $T_\theta(\mu) = \widetilde{\mu}$ where T_θ denotes rotation through angle θ.)

3. (10 points) a) Did you find the portfolio assignment worthwhile? Why or why not?

b) What suggestions do you have for improving the portfolio assignment in the future.

(**Note:** Full credit will be given for a thoughtful answer on this question.)