Math 436
December 11, 2000

FINAL

Show all your work. If you find you are doing a horrendous calculation, you are making a mistake or at least making the problem unnecessarily difficult. (Some calculations are necessary.)

1. (25 points) a) Solve:

$$u_{tt} = u_{xx}, \qquad t > 0, \ 0 < x < 1,$$
$$u(x,0) = 0, \qquad 0 < x < 1,$$
$$u_t(x,0) = x(1-x), \qquad 0 < x < 1,$$
$$u(0,t) = 0 = u(1,t), \qquad t > 0.$$

b) Show that your answer solves the problem.

c) Find $u(\frac{3}{4}, \frac{5}{4})$. (Your answer should be a number.)

2. (10 points) Find the solution $u(\rho, \phi)$ of Laplace's equation in the disk $\rho < 1$ and satisfying the boundary condition

$$u(1, \phi) = 54917 \sin 6327145\phi - 72901 \cos 143587\phi.$$

3. (20 points) a) Solve:

$$u_t = \nabla^2 u, \qquad 0 < x < \pi, \ 0 < y < \pi, \ 0 < t,$$
$$u(0,y,t) = 0 = u(\pi,y,t) = u(x,0,t) = u(x,\pi,t),$$
$$u(x,y,0) = xy(\pi - x)(\pi - y).$$

b) Show that your answer solves the problem.

4. (20 points) a) Solve:

$$u_t = u_{xx}, \qquad t > 0, \ -\infty < x < \infty,$$
$$u(x,0) = \begin{cases} 0, & x < -5 \\ x, & -5 < x < 8 \\ 0, & 8 < x. \end{cases}$$

b) Find $u(0,t)$. (Your answer should be a function of t which does not involve an integral.)

c) What is $\lim_{t \to 0^+} u(-5,t)$?