Math 437 **Fall semester 1997-98**

Review sheet for Test 2.

1. **Affine transformations**. Affine reflections. Similarities. Shear transformations. Affine transformations preserve line, the ratio of three collinear points. An affine transformation is determined by three non-collinear points an their images.

Sample questions: **1**.1 Find the affine transformation, which maps: (0, 0) --> (1, 1), (1, 1) --> (1, 0), (1, -1) --> (0, 1). Is the transformation, an affine reflection, a similarity, a shear transformation or neither.

1.2 Show that if an affine transformation maps P --> Q and Q --> P, then the midpoint M of the segment P, Q is a fixed point of the transformation. Show that this information suffices to find the image of every point of the line PQ. If the ratio $(P, Q; R) = q$ what is the ratio $(P, Q; R')$, where R' is the image of R. Can you tell what is the image of a point off the line? Provided the line RR' is parallel to PQ is R the image if R'.

2. **Rays and angles**. Definition of a ray. Definition of an angle. Measure of an angle. The measure of an angle is preserved by isometries and similarity transformations but not by general affine transformations. Straight angles, right angles, acute and obtuse angles. Supplementary angles, vertical angles. Angles and rotations. Angles of the same measure are congruent. The fifth Euclid's postulate. The interior of an angle. The crossbar theorem. Addition theorem for angles.

Sample questions: **2**.1. Let $P = (4, 4)$, $Q =(1, 0)$, $R = (-3, 3)$. Find the unit directional vectors of the arms of the angle PQR. Find the measure of the angle. Is the angle acute, right, obtuse, straight? Is the point (0, 0) an interior of exterior point of the angle. Is the point (2, 0) an interior point of the angle.

2.2. State and prove the crossbar theorem.

2.3. State and prove the addition theorem for angles.

3. **Triangles**. The interior of the triangle. Isosceles triangles. The sum of angles of a triangle. Congruence criteria for triangles. The cosine theorem. The sine theorem.

Sample questions: **3**.1. $A = (-1, -1)$, $B =(3, -1)$, $C = (2, 1)$. Is the origin (0, 0) in the interior of the triangle ABC?

3.2. Prove the for any interior point X of a triangle PQR the distance of X to the line carrying a side of the triangle is smaller than the distance of the opposite vertex from the line.

3.3 Prove that the sun of angles of a triangle equals 2π.

3.4. State and prove the cosine and sine theorems.

4. **Circles**: Definition of a circle. Equation of a circle. Intersection of a circle and a line and of two circles. The power of a point relative to a circle. Interior and exterior points of a circle. The power axes of two circles. The power center of three circles.

The measure of angles inscribed in a circle.

Sample questions: **4**.1. Find a point whose power relative to the circles C_1, C_2, C_3 is the same (this point is called the power center of the three circles), The circles have centers (0, 0), (2, 0), (0, 2) respectively and are all of radius 1.

4.2. Prove that a line through an interior point of a circle, has two intersection points with the circle.

4.3. Prove that there are two lines through an exterior point of a circle, which have with the circle exactly one point in common. Prove that each of those lines is perpendicular to the line connecting the common point with the center of the circle. Use the result to justify the following construction of a tangent to a given circle through a given exterior point: Draw a circle through the given point with center at the midpoint of the segment, whose endpoints are the given point and the center of the given circle. The two tangents go through the given point and through the intersection points of the two circles.