Math 437 **Fall semester 1997-98**

Review of material covered after the second test

The projective plane. Homogeneous coordinates. Point coordinates and line coordinates, duality.
The projective plane and the affine plane, the improper line or line at infinity.
Projective collineations. A projective collineation is determined by 4 points, with no three of them collinear, and their images. Perspectivities, center and axis of a perspectivity., homologies and elations.
Cross ratio of four collinear points. Invariance of cross ratio under projective collineations. Cross ratio versus simple ratio in the affine plane. Harmonic points.

Sample questions:

1. How many different points of the projective plane are represented by the following homogeneous coordinates? Are any three of them collinear? Any four?

   (1, 0, 1), (1, 1, 1), (2, 0, 2), (3, 3, 3), (1, 0, 2), (2, 1, 2), (3, 1, 3).

2. How many different lines are determined by pairs of these points?

3. Find the homogeneous line coordinates of these lines. Which point lies on which line.

4. If the coordinates in 1 are line coordinates rather than point coordinates, is there a set of concurrent lines larger than two.

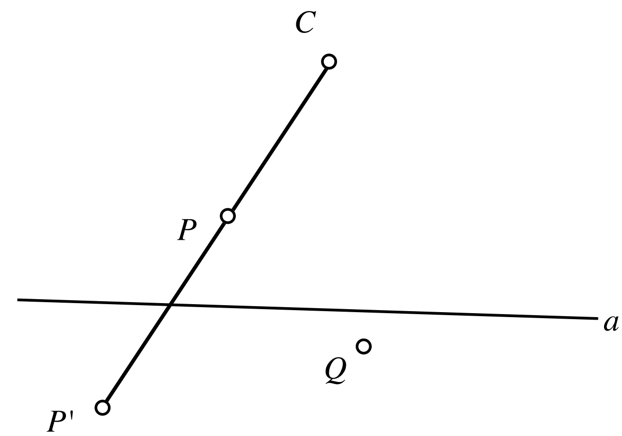
5. Find the homogeneous coordinates of the intersection point of the following pairs of lines of the affine plane in the augmentation to a projective plane.

   $2x - y = 2$ and $x - 2y = 3$, $2x + y = 3$ and $2x + y = 4$.

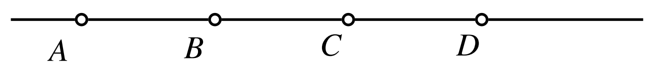
6. Write the equations of the above line in homogeneous coordinates.

7. A projective collineation maps the line $x_0 = 0$ into the line $x_2 = 0$, leaving the common point (1, 0, 0) invariant. Find the matrix of the most general transformation of this type.

8. In the picture below, C is the center and a the axis of a perspectivity, Knowing that the image of P is P' , construct the image of Q.

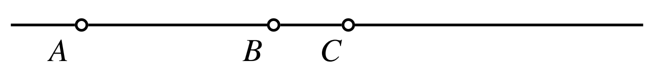


9. The points A, B, C, D are collinear and with equal distances between consecutive points.



Find the cross ratios $(A,B;C,D)$ and $(A,C;B,D)$.

10. In the figure below construct the fourth harmonic to B with respect to A and C.



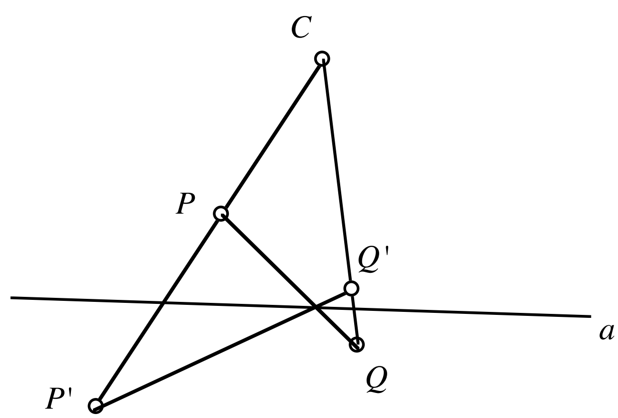
Answers

1. 5 points: A= [(1,0,1)]=[(2,0,2)], B=[(1,1,1)]=[(3,3,3)], C=[(1,0,2)], D=[(2,1,2)], E=[(3,1,3)].
2. Points A,B,D,E are collinear, so there are 5 lines $\overset{\times}{AB}$, $\overset{\times}{CA}$, $\overset{\times}{CB}$, $\overset{\times}{CD}$, $\overset{\times}{CE}$
3. The homogeneous cordinate of these lines are respectively: (1,0,-1), (0,1,0),(2,-1,-1), (2,-2,-1), (2,-3,-1).
4. Yes the lines with coordinater (1,0,1), (1,1,1), (2,1,2) and (3,1,3) are concurrent.

5. (1,-4,3), (1,-2,0). 6. $2x_1 - x_2 - 2x_0 = 0$, $x_1 - 2x_2 - 3x_0 = 0$, $2x_1 + x_2 - 3x_0 = 0$, $2x_1 + x_2 - 4x_0 = 0$.

7. $\lambda \begin{bmatrix} 1 & a & b \\ 0 & 0 & c \\ 0 & d & e \end{bmatrix}$, where λ, a, b, c, d, e are arbitrary constants and $cd \neq 0$.

8.



9. $(A,B;C,D) = \frac{3}{2}$, $(A,C;B,D) = -3$.

10.