

Peter Cholak Math 441 Friday, October 18 Exam 1

This exam is worth 100 points. Be sure to prove your answers are correct.

The deal: If you choose you can rewrite the exam over the break (due in the beginning of class the Monday after break – no exceptions). In which case, your score for the exam will be the maximum of the score of how you did in class or the average of the two scores. Since you will have more time for the second write-up, I will expect more complete answers and this will be reflected in it's grading.

(10 points) Find a regular expression α such that $\{aa\}^* \cap L(\alpha) = \emptyset$ and $L(\alpha)$ is infinite. What is $L(\alpha)$?

(15 points) Find a finite automaton such that $L(M) = \overline{\{bb\}^* \cup \{aa\}^*}$.

(20 points) Let M be *any* finite automaton. Give an algorithm for determining whether $|L(M)| = 1$

(20 points) State some form of the Pumping Lemma and use it to show $\{a^n b^m : m \leq n \leq 2m\}$ is not regular.

(10 points) What is the definition of a regular grammar. It is not possible to find a regular grammar G such that $L(G) = \{a^n b^m : m \leq n \leq 2m\}$. Why? (You may assume the last problem.)

(25 points) Find a context-free grammar G such that $L(G) = \{a^n b^m : m \leq n \leq 2m\}$. (One direction of the proof that your grammar is correct will involve an induction on the length of the derivations.)