Peter Cholak Math 441 Friday, December 20 Final

This exam is worth 200 points. Be sure to prove your answers are correct! Problems 2a,b,c and 4c are the hardest problems on the exam (or so I think). If you have not looked at these before I suggest dealing with them at the end of the exam. Otherwise the problems are listed in the order we covered them.

Let $M = (\{s, p, q, f\}, \{a, b\}, \{(s, a, p), (p, b, f), (p, b, q), (f, a, p), (q, a, f)\}, s, \{f\})$. (5 points) M is a nondeterministic finite automata. Why? (10 points) Draw the state diagram for M. (10 points) Give a regular expression for the language accepted by M.

(Parts of 2.5.6) An arithmetic progression is the set $\{p+qn : n \in \mathbb{N}\}$ for some $p, q \in \mathbb{N}$. (10 points) Show that if $L \subseteq \{a\}^*$ and $\{n : a^n \in L\}$ is an arithmetic progression then L is regular. (10 points) Show that if $L \subseteq \{a\}^*$ and $\{n : a^n \in L\}$ is a union of finitely many arithmetic progressions then L is regular. (20 points) Show that if $L \subseteq \{a\}^*$ is regular then $\{n : a^n \in L\}$ is a union of finitely many arithmetic progressions.

(15 points) Find an context-free grammar G such that $L(G)$ is not regular.

(Parts of 3.5.28) Which of the following languages are context-free? Explain briefly in each case. (5 points) $\{a^m b^n c^p : m = n or n = p or m = p\}$ (10 points) $\{w \in \{a, b, c\}^* : w does not contain equal numbers of a, b, and c$ (20 points) $\{w \in \{a, b\}^* : w = w_1 \dots w_m for some m \geq 2 and some w_1, \dots w_m such that |w_1| = \dots = |w_m| \geq 2\}$ (Hint: is $\Psi(L)$ semi-linear?)

(25 points) Build a Turing machine which *accepts* $\{w \in \{a, b\}^* : |w| is odd\}$.

(25 points) (Essay) What is Church's thesis? Do you believe it? Why or Why not?

(25 points) Given an example of an infinite language which is not context-free but is Turing decidable (use Church's thesis to show this language is Turing-decidable).

(10 points) Fix a Turing Machine M. Is there an algorithm which decides if $a^{179}b^{213} \in L(M)$?