Exam. III, Math. 441, Fall, 1998   Your name:

This exam. consists of 10 questions. For certain questions, there is little or no computation, and there can be no credit for an incorrect answer. However, on questions where it is not possible to write down the answer without doing some preliminary steps, partial credit may be given, and you should be sure to show your work. The numbered spaces below are not for answers, but for scores.

problem	points
1.	
2.	
3.	
4.	
5.	
6.	
7.	
8.	
9.	
10.	

1. Complete the following definition of acceptance for pushdown automata:
Let M = (Q,Σ,Γ,Δ,s,F) be a pushdown automaton, and let w∈Σ*.

Then w∈L(M) if and only if (s,w,e) $\vdash_M^*$ (r, ,), where

[Fill in the blanks in the final configuration, and say what must be true of r.]

2. Consider the pushdown automaton M = (Q,Σ,Γ,Δ,q,{r}), where Q = {q,r}, Σ = {a,b}, Γ ={a}, and Δ = {((q,a,e),(q,a)), ((q,e,e),(r,e)), ((r,b,a),(q,e))}. Show that aabb∈L(M), by completing the sequence of configurations started below.

(q,aabb,e) $\vdash_M$

3. For the pushdown automaton in Problem 2, what is L(M) ?

(a) {w∈{a,b}*:w has at least one a and at least one b}
(b) {w∈{a,b}*:w has even length}
(c) {w∈{a,b}*:w has equal numbers of a's and b's}
(d) a*∪b*
(e) $\{a^n b^n : n \in N\}$

4. The language $L = \{a^r b^r a^r : r \in N\}$ is not context-free. Complete the proof of this below, using the Pumping Theorem for Context-Free Languages.

Suppose L is context-free. Let n be as in PTCFL. Choose $w = a^n b^n a^n$. Then $w \in L$, $|w| \geq n$. Therefore, there exist u,v,x,y,z such that

(i)

(ii)

(iii) for all k,

Explain why v is in a* or b*, and the same for y.

Now, choose an appropriate k to arrive at a contradiction, so that you can conclude that L is not context-free.

5. Suppose L and L' are context-free languages over the alphabet {a,b}. Which of the following must be context-free ? [There may be more than one.]
(a) $L \cup L'$
(b) $L \cap L'$
(c) $\{w \in \{a,b\}^* : w \notin L\}$

6. Consider the Turing machine $M = (Q,\Sigma,\delta,q_0,\{q_2\})$, where $Q = \{q_0,q_1,q_2\}$, $\Sigma = \{\triangleright,\sqcup,a,b\}$, $\delta(q_0,\triangleright) = \delta(q_0,\sqcup) = \delta(q_0,b) = (q_0,\rightarrow)$, $\delta(q_0,a) = (q_1,\rightarrow)$, $\delta(q_1,\triangleright) = \delta(q_1,\sqcup) = \delta(q_1,a) = (q_1,\rightarrow)$, $\delta(q_1,b) = (q_2,\rightarrow)$. Complete the computation of M on <u>input</u> aabaa--so the initial configuration is the one given below.

$(q_0,\triangleright\underline{\sqcup}aabaa) \vdash_M$

7. Let M be as in Problem 6. What is the set of strings $w\in\{a,b\}^*$ such that M will eventually halt, given input w--initial configuration $(q_0,\triangleright\underline{\sqcup}w)$?

(a) $\{w\in\{a,b\}^*$:w has at least one a and at least one b$\}$

(b) a^*b^*

(c) $\{w\in\{a,b\}^*$:w has at most one b$\}$

(d) $\{a,b\}^*$

(e) $\{w\in\{a,b\}^*$:w has ab as a substring$\}$

8. Suppose C is a Turing machine which transforms any tape configuration of form $\triangleright\underline{\sqcup}1^n$ into $\triangleright\underline{\sqcup}1^n\sqcup1^n$. How does the composite machine M below transform the tape configuration $\triangleright\underline{\sqcup}11$?

$>CR \qquad L$

$R_{\sqcup}1R_{\sqcup}L\sqcup L_{\sqcup}$

(a) $\triangleright\underline{\sqcup}11$ (b) $\triangleright\underline{\sqcup}111$ (c) $\triangleright\underline{\sqcup}1111$ (d) $\triangleright\underline{\sqcup}11111$ (e) M does not halt, given this input

9. Let M be as in Problem 8. What numerical function does M compute ? [Recall that M computes f if it transforms the tape configuration $\triangleright\underline{\sqcup}1^n$ into the tape configuration $\triangleright\underline{\sqcup}1^{f(n)}$.]

(a) $f(n) = n+1$

(b) $f(n) = n+3$

(c) $f(n) = 2n$

(d) $f(n) = n^2$

(e) M does not compute any numerical function

10. Which of the following statements are true ? [There may be more than one.]

(a) If M is a finite automaton, then L(M) is context-free.

(b) If G is a context-free grammar, there is a finite automaton M such that L(M) = L(G).

(c) If G is a context-free grammar, there is a pushdown automaton M such that L(M) = L(G).

(d) If M is a finite automaton, there is a Turing machine which decides L(M).