Exam. I, Math. 441, Fall, 1998 Your name:

This exam. consists of 10 questions. For certain questions, there is little or no computation, and there can be no credit for an incorrect answer. However, on questions where it is not possible to write down the answer without doing some preliminary steps, partial credit may be given, and you should be sure to show your work. The numbered spaces below are not for answers, but for scores.

problem	points
1.	
2.	
3.	
4.	
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9.	
10.	

1. The non-deterministic finite automaton M shown on the left accepts the language represented by the regular expression a*ab. Applying to M the general method for transforming a non-deterministic finite automaton into an equivalent deterministic one (and dropping the non-reachable states), we obtain the machine N with state diagram shown on the right.

Complete the set-theoretic description of the states of N.

$s_0 = \{q_0\}$
$s_1 =$
$s_2 =$
$s_3 =$

2. Let L = L(M), where M is the deterministic finite automaton shown below on the left. Which of the non-deterministic finite automata on the right accepts the language L* ?

3. Let L = $\{a^n b a^{2n} : n \in N\}$. Show, using the Pumping Theorem, that L is not regular.

4. Let L be the language from Problem 4, and let $\approx_L$ be the equivalence relation on $\{a,b\}^*$ such that $x \approx_L y$ if and only if $\forall z\ xz \in L \Leftrightarrow yz \in L$. Show that a ♠$_L$ aa, by giving some string $z \in \{a,b\}^*$ such that $az \in L$ and $aaz \notin L$.

5. Let L = $\{a^{m+1}ba^nba^{2n}: m,n \in N\}$. Note that for any string $w \in L$, we can express w as xyz, where x = e, y = a, and for all $k \in N$, $xy^kz \in L$. Which result would you try to use to determine whether L is regular ?

(a) Pumping Theorem

(b) Myhill-Nerode Theorem

(c) other--specify

6. Let L = $\{a^mb^n: m,n \in N\}$.

For the strings e, a, b, ab, ba, determine which represent new equivalence classes under $\approx_L$; i.e., which of the following are true (there may be more than one).

(a) a $\spadesuit_L$ e

(b) b $\spadesuit_L$ e & b $\spadesuit_L$ a

(c) ab $\spadesuit_L$ e & ab $\spadesuit_L$ a & ab $\spadesuit_L$ b

(d) ba ab $\spadesuit_L$ e & ba $\spadesuit_L$ a & ba $\spadesuit_L$ b & ba $\spadesuit_L$ ab

7. For the language L of problem 6, how many equivalence classes are does $\approx_L$ have in all ?

(a) 2 (b) 3 (c) 4 (d) 5 (e) infinitely many

8. Consider the context-free grammar G = (V,S,R,S), where V = {a,b,S}, S = {a,b}, R = {(S,aSaa), (S,b)}. Give a derivation of the string aabaaaa. Take care that each step represents a single application of a rule.

9. Give a parse tree corresponding to the derivation in Problem 8 (a).

10. Let L be the language represented by the regular expression a^*b^*.

(a) Design a context-free grammar $G = (V,\Sigma,R,S)$ such that $L(G) = L$.

(b) Check your answer to part (a) by giving derivations of e, a, b, and aabb.