Exam. I, Math. 441, Fall, 1998 Your name:

This exam. consists of 10 questions. For certain questions, there is little or no computation, and there can be no credit for an incorrect answer. However, on questions where it is not possible to write down the answer without doing some preliminary steps, partial credit may be given, and you should be sure to show your work. The numbered spaces below are not for answers, but for scores.

problem	points
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1. Let f be the function from natural numbers to strings on {a,b} such that f(0) = a and if f(n) is the string x, then f(n+1) = bxb. Show that for all n, f(n) has length 2n+1. [Use induction on n.]

2. Match the regular expressions (i)-(v) with the languages they represent (the expressions are missing some parentheses).

(i) $(ab)*\cup(ba)*$
(ii) $((a\cup\varnothing*)b)*(a\cup\varnothing*)$
(iii) $((a\cup b)(a\cup b))*$
(iv) $(b\cup\varnothing*)(ab)*(a\cup\varnothing*)$
(v) $(a*b*a*)*$

(a) $\{w\in\{a,b\}^*$:w does not have aa as a substring$\}$
(b) $\{w\in\{a,b\}^*$:w does not have either aa or bb as a substring$\}$
(c) $\{w\in\{a,b\}^*:|w|$ is even$\}$
(d) $\{w\in\{a,b\}^*:|w|$ is even and w does not have either aa or bb as substring$\}$
(e) $\{a,b\}^*$

3. Which of the following pairs of regular expressions represent the same language (again some parentheses are missing) ? In case the languages are different, give an example of a string which is in one and not the other.

(a) $((aa\cup a)b)^*$; $(aab)^*\cup(ab)^*$

(b) $(a\cup b)^*$; $(a^*b^*)^*$

(c) $\varnothing$; $\varnothing^*$

4. Consider the deterministic finite automaton (Q,Σ,δ,s,F), where $Q = \{s,f\}$, $\Sigma = \{a,b\}$, $F = \{f\}$, and $\delta(s,a) = f$, $\delta(s,b) = s$, $\delta(f,a) = s$, $\delta(f,b) = f$. Which of the following strings are accepted--there may be more than one ?

(a) e (b) bab (c) babbab (d) aaa (e) baaab

5. For the deterministic finite automaton M from Problem 4, what is L(M) ?

(a) $\{w \in \{a,b\}^*$:w has at least one a$\}$

(b) $\{w \in \{a,b\}^*$:w has an even number of a's$\}$

(c) $\{w \in \{a,b\}^*$:w has an odd number of a's$\}$

(d) $\{w \in \{a,b\}^*$:each a in w has b's on either side side$\}$

(e) $\{w \in \{a,b\}^*$:w has equal numbers of a's and b's$\}$

6. For the deterministic finite automaton M with the state diagram below, complete the sequence of configurations showing that abab$\in$L(M). Call the initial state s and the other state q.

b b

a

> ◎ o

a

(s,abab) $\vdash_M$ $\vdash_M$ (s,e)

7. Complete the following definition. Let M = (Q,Σ,Δ,s,F) be a nondeterministic finite automaton, and let q,r$\in$Q, u,v$\in\Sigma^*$. Then (q,u) $\vdash_M$ (r,v) if (and only if) either

(i) there exists a$\in$Σ such that u = av

and ____ or

(ii) u = v

and ____ .

8. Consider the nondeterministic finite automaton $M = (Q,\{a,b\},\Delta,s_0,F)$, where $Q = \{s_0,s_1,s_2\}$, $F = \{s_2\}$, and $\Delta = \{(s_0,a,s_1), (s_0,a,s_2), (s_1,b,s_2), (s_2,b,s_0)\}$.

(a) Give a sequence of configurations showing that $(s_0,ab) \vdash^*_M (s_2,e)$.

(b) Give a sequence of configurations showing that $(s_0,ab) \vdash^*_M (s_0,e)$.

(c) Say whether $ab \in L(M)$.

9. Consider the non-deterministic finite state automaton $M = (Q,\Sigma,\Delta,s_0,F)$, where $Q = \{s_0,s_1,s_2\}$, $F = \{s_1,s_2\}$, $\Sigma = \{a,b\}$, and $\Delta = \{(s_0,e,s_1),(s_0,e,s_2),(s_1,a,s_1),(s_2,b,s_2)\}$.

(a) Draw a state diagram for M.

(b) Say whether $ab \in L(M)$.

10. Which regular expression (with missing parentheses) represents the language accepted by the nondeterministic finite automaton whose state diagram is given below ?

>o a o a o b Ⓞ

a a

o

b a

Ⓞ

(a) aab(aab)*∪ab(ab)* (b) ((aa∪a)b)* (c) ((aa∪a)b)(ab)* (d) (aa∪a)b* (e) Ø*