Final Exam., Math. 441, Fall, 1998   Your name:

This exam. consists of 20 questions. For certain questions, there is little or no computation, and there can be no credit for an incorrect answer. However, on questions where it is not possible to write down the answer without doing some preliminary steps, partial credit may be given, and you should be sure to show your work. The numbered spaces below are not for answers, but for scores.

problem	points
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The following finite automaton is used in Problems 1-2.
$M = (Q,\Sigma,\delta,q_0,\{q_0\})$, where $Q = \{q_0,q_1\}$, $\Sigma = \{a,b\}$, and δ is given in the table below

q	σ	$\delta(q,\sigma)$
q_0	a	q_1
q_0	b	q_0
q_1	a	q_0
q_1	b	q_1

1. Which of the following strings are in L(M) ?

(a) e (b) a (c) ab (d) baba (e) bbb

<u>Hint</u>: It may be helpful to draw a state diagram.

2. Which one of the following expressions represents L(M) ?

(a) $(a \cup b)^*$ (b) $b^*a \cup a^*b$ (c) $b^*ab^*ab^*$ (d) $b^*(ab^*ab^*)^*$ (e) $b^*(ab^*)^*$

3. Complete the following definition.
A <u>non-deterministic finite automaton</u> is a tuple $M = (Q,\Sigma,\Delta,s,F)$, where Q and Σ are finite sets, and

(i) $s \in$

(ii) $F \subseteq$

(iii) $\Delta \subseteq$

4. The general procedure for transforming a non-deterministic finite automaton into an equivalent deterministic finite automaton transforms M, with state diagram shown on the left, into N, with state diagram shown on the right. Complete the description of the states of N below.

M			N				
a,b		a,c	a,b		a,c		a,b,c
>o	e	◎	>◎	c	◎	b	o
q_0		q_1	s_0		s_1		s_2

$s_0 = \{q_0,q_1\}$

$s_1 =$

$s_2 =$

5. Which of the following classes of languages are closed under concatenation ?

(a) regular

(b) context-free

(c) semidecidable

(d) decidable

(e) polynomially decidable

6. Which of the following classes of languages are closed under intersection ?

(a) regular

(b) context-free

(c) semidecidable

(d) decidable

(e) polynomially decidable

7. The language $L = \{ww^R : w \in \{a,b\}^*\}$ is not regular. Complete the proof of this fact below.

Suppose L is regular. Let n be as in the Pumping Theorem (for regular languages). Let $w = a^n bba^n$. Then there exist x, y, z such that

(i)

(ii) and

(iii) for all $k \in N$,

Now, choose an appropriate k so that you can arrive at a contradiction and conclude that L is not regular.

8. Suppose L is a language such that $\approx_L$ has infinitely many equivalence classes. What can you conclude [more than one may hold] ?

(a) L is not regular

(b) L is not context-free

(c) L is not semidecidable

(d) the complement of L is not regular

(e) L is not decidable

The language L = {w∈{a,b}*:w has aba as a substring} is used in Problems 9-10.

9. For each of the following pairs of strings x and y, show that $x \spadesuit_L y$ by giving some z such that xz∈L and yz∉L, or vice versa.

(a) x = e and y = a

(b) x = a and y = ab

(c) x = ab and y = aba

10. How many equivalence classes are there ?

(a) 3 (b) 4 (c) 5 (d) 6 (e) infinitely many

The following context-free grammar is used in Problems 11-12.

$G = (V,\Sigma,R,S)$, where $V = \{S,A,B,a,b\}$, $\Sigma = \{a,b\}$,
$R = \{S \rightarrow ASB, S \rightarrow AS, A \rightarrow a, B \rightarrow b, S \rightarrow e\}$.

11. Complete a derivation showing that the string $aab \in L(G)$.

$S \Rightarrow_G$

12. What is $L(G)$?

(a) $a^*(ab)^*$ (b) a^*b^* (c) $\{w \in \{a,b\}^*$: w has at least as many a's as b's$\}$

(d) $\{a^n b^n : n \in N\}$ (e) $\{a^m b^n : m \geq n\}$

The following Turing machine is used in Problems 13-14.
$M = (Q,\Sigma,\delta,q_0,\{h\})$, where $Q = \{q_0,q_1,q_2,q_3,h\}$, $\Sigma = \{\triangleright,\sqcup,a,b\}$, and
δ is given in the table below (with some omissions).

q	s	$\delta(q,s)$
q_0	$\sqcup$	$(q_1,\rightarrow)$
q_1	a	(q_2,b)
q_1	b	$(q_2,\sqcup)$
q_1	$\sqcup$	$(q_3,\leftarrow)$
q_2	b	$(q_1,\rightarrow)$
q_2	$\sqcup$	$(q_1,\rightarrow)$
q_3	a	$(q_3,\leftarrow)$
q_3	b	$(q_3,\leftarrow)$
q_3	$\sqcup$	$(q_3,\leftarrow)$
q_3	$\triangleright$	$(h,\rightarrow)$

13. Complete the computation on input abab

$(q_0,\triangleright\underline{\sqcup}abab) \vdash_M$

14. Describe in general how M transforms the tape configuration $\triangleright\underline{\sqcup}w$, for $w\in\{a,b\}^*$.

15. Which of the following statements are true of the language $L = \{1^{n^3} : n \in N\}$?

(a) L is regular

(b) L is context-free

(c) L is semidecidable

(d) L is decidable

Hint: You may use the fact that if $r \neq 0$, then for some k, s+kr is not a perfect cube.

16. Suppose M computes the function $f(x) = x+1$, and N computes the function $g(x) = x^2$, both with inputs and outputs in unary. Which machine computes the function $h(x) = (x+1)^2+1$?

(a) >MNN (b) >MMN (c) >MNM (d) >NMM (e) >NMN

17. Let H = $\{1^n : M_n$ eventually halts, given input $1^n\}$. Which of the following statements are true of H ?

(a) H is decidable

(b) H is semidecidable

(c) the complement of H is semidecidable

(d) H is in P

(e) the complement of H is in P

18. State the Church-Turing Thesis.

19. Complete the following definition of the class P.

A language $L \subseteq \Sigma^*$ is in P if there exist a Turing machine M, and a polynomial p(x) (with integer coefficients) such that M decides L, and for $w \in \Sigma^*$ of length n,

20. Let $E = \{1^n : M_n$, given input 1^n, halts in $\leq 2^n$ steps$\}$. Which of the following statements are true of E ?

(a) E is decidable

(b) E is semidecidable

(c) the complement of E is semidecidable

(d) E is in P

(e) the complement of E is in P