Math 441, Computability and Logic

Midterm Exam 1, Spring 2000

REMARK: QUOTE ALL THEOREMS YOU ARE APPLYING.

1.a) Define a regular expression over an alphabet $\sum$.
1.b) Suppose that $L \neq \sum^*$ is a regular language over an alphabet $\sum$. Is it true that there is necessarily a regular language $K \neq \sum^*$ such that $L \subseteq K$ and $K \neq L$? Prove your answer.

2) Find a regular expression for the following languages (where, $\sum = \{a, b\}$):
(i) $L_1 = \{w \in \sum^* \mid$ whenever $i < j \leq |w|$ and $w(i) = a$ and $w(j) = a$ and for all k with $i < k < j$, $w(k) = b$ the difference $j - i$ is odd $\}$
(ii) $L_2 = \{w \in \sum^* \mid w$ does not contain the string aaa as substring $\}$

3) Find a deterministic finite automaton which accepts only words in the following language over $\sum = \{a_1, a_2, a_3\}$:
$L = \{w \in \sum^* \mid$ for some $1 \leq i \leq 3$, a_i does not appear in w $\}$

4) Let L be a regular language over $\sum$. Prove that the language $K = \bigcup_k (\sum^* \setminus L)^{3k+1}$ is accepted by a deterministic finite automaton.

5) What is the language accepted by the following deterministic finite automaton (you may either characterize this language or describe it by a regular expression) ?