

Math 441, Computability and Logic

Final exam, Spring 2000

A GENERAL REMARK: QUOTE ANY THEOREM THAT YOU ARE MAKING USE OF.

Part I

ANSWER 4 QUESTIONS, GIVE A SHORT EXPLANATION TO YOUR ANSWER.

- 1) Define the notion of a context-free language. Is it true that every regular language is a context-free language ?
- 2) Define the notion of a push-down automaton.
- 3) Prove or refute: If L is a context-free language then the number of equivalence classes of \approx_L is finite.
- 4) Prove or refute: If L and K are regular languages then $LK \cap KL$ is a context-free language.
- 5) Prove or refute: there is a push-down automaton M which accepts the language $L = \{0^n 1^n \mid n \text{ is a natural number}\}$.

Part II

ANSWER 4 QUESTIONS.

- 1) Prove that the language $L = \{w \in \{a, b\}^* \mid \text{the number of } a\text{-s in } w \text{ is not divisible by } 3\}$ is regular.
- 2) Construct a push-down automaton which accepts the following language: $L = \{w \in \{a, b, c\}^* \mid w = xc^n x^R \text{ for some natural number } n \text{ and } x \in \{a, b\}^*\}$.
- 3) Find a regular expression for the language $L = \{w \in \{a, b\}^* \mid w \text{ has odd number of } b\text{-s}\}$.
- 4) Prove that if L is a regular language and $a_0, a_1, \dots, a_n, \dots$ is an infinite arithmetic progression of natural numbers then $\bigcup_i L^{a_i}$ is regular.
- 5) Let $L = \{w \in \{a, b\}^* \mid \text{the number of } a\text{-s appears in } w \text{ is bigger than twice the number of } b\text{-s appears in } w\}$. Show that L is not regular.

GOOD LUCK !!