Math 468: Topology
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J. Palmieri

Topology is the study of shapes; roughly speaking, two shapes are "topologically equivalent" if one can be deformed continuously into the other. For instance, a sphere is topologically equivalent to a cube:

On the other hand, a sphere is not topologically equivalent to a torus (although it's hard to give a good reason without knowing some topology).

We will start the course by looking at the basic ideas of topology—topological space,[1] continuity, compactness, and connectedness—in order to make the notion of topological equivalence more precise, and in order to develop some intuition for topological ideas.[2]

Next, we will move on to differential topology. Given a function between two nice topological spaces (a.k.a. "manifolds"), we will define the derivative of that function (if it exists). It is *not* possible to deform a sphere into a cube in a differentiable way, because of the corners. Using the derivative, we will define and discuss other topics, including: transversality, intersection theory, winding numbers, and Euler characteristic. At some point, we will develop enough tools to be able to say precisely why the sphere is not topologically equivalent to the torus.

PREREQUISITES: The students should have a basic background in analysis (advanced calculus), linear algebra, and perhaps a bit of abstract algebra.[3] In particular, they should know and be able to work with the definition of continuity and the definition of derivative, and they should be comfortable working with matrices and linear transformations.

TEXTBOOK: *Differential Topology* by Victor Guillemin and Alan Pollack.

[1] The definition of a topological space is rather abstract; for this course, we will work almost exclusively with subsets of Euclidean space $\mathbf{R}^n$ (where one's intuition is a bit more reliable).

[2] I'm hoping that most students will have at least seen the words "continuous" and "compactness" before. Regardless, we will discuss these ideas thoroughly.

[3] In particular, a little group theory.