

18 Topology  
Spring 1999  
John Palmieri

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Text: A combinatorial introduction to topology, by Michael Henle.

Topics covered:

Chapter 1, "Basic Concepts"

intro  
continuity, closed sets, open sets  
compactness, connectedness

Chapter 2, "Vector Fields"

Brouwer fixed point theorem  
indices of critical points  
Poincare index theorem

Chapter 3, "Plane Homology and the Jordan Curve Theorem"

statement of Jordan curve theorem  
chains, cycles, boundaries, homology in the plane  
proof of Jordan curve theorem

Chapter 4, "Surfaces"

combinatorial definition of surface  
classification theorem

Chapter 5, "Homology of Complexes"

homology groups of a complex  
invariance

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I relied pretty heavily on the book in this course, so I'll make some comments on it: first, it's put out by Dover, so it's cheap (around \$10). Second, it covers a nice collection of topics: by the end of the semester, we had done the Brouwer fixed point theorem, the Jordan curve theorem, the classification of surfaces, and we had defined (mod 2) homology. The book uses a combinatorial approach, and has some clever arguments that make the material elementary and accessible. The proof of the Brouwer fixed point theorem is a good example: we were able to do this within the first three weeks of the course.

For several reasons, the book might not be good for students who need to know abstract topology because they're going on to graduate school: most all of the topology is done with subsets of the plane, and so you can avoid the general definition of open, closed, continuous, compact, etc. Also, the book uses somewhat nonstandard language: the

standard topological concepts are defined in terms of points being "near" sets. This nonstandard language, by the way, is what prompted me to give them the handout `standard.tex'. Of course, if the students are good enough to go on to graduate school, they ought to be able to fill in the gaps by reading a more serious introduction, like Munkres' book. For students not going on to graduate school, I think the book is quite appropriate.)

The reason I relied so heavily on the book is that I hardly lectured at all. Every class, I assigned reading and homework problems (both of which are pretty good in this book); we spent class time going over questions on the reading and the homework. Mostly, I acted as a moderator: the students would suggest their solutions to problems, and I would write them on the board. Occasionally, the students would put their own work on the board, too. Twice during the semester, I had the students hand in "portfolios" of their homework problems; here is how I described the assignment in class (and also what I posted on the web page):

Turn in your portfolio of homework problems for the semester so far (actually, up to and including Section 15).

I am looking for an indication that you have tried the assigned problems, and that either you solved them yourself or you learned (and recorded) something from the class discussion of the problem. Or both.

The perfect portfolio would be organized and would have complete, well-written solutions to all of the problems so far; it could be published as part of a solution manual for the book.

The good portfolio would be organized and have well-written, almost complete solutions to almost all of the problems.

Skipping a few levels in quality, the barely acceptable portfolio would have something for most of the problems; the something would be relevant, but perhaps scrawled on the backs of envelopes or those paper placemats from Chinese restaurants.

The almost acceptable portfolio would have something relevant for about half of the assigned problems.

This approach seemed to work pretty well. Some students worked hard on the problems and solved them before class, some of them worked but didn't solve them (and then incorporated the solutions discussed in class into their portfolios), and one or two of them seemed a bit lazy and they had to work really hard to try to come up with an acceptable portfolio). I was also willing to give lots of help on the homework,

both in class and in my office, so there wasn't much excuse for turning in a bad portfolio.

In the end, I based the grade on the portfolio and the final exam, so they were rewarded for hard work (the portfolio) and knowledge (the exam).

A disadvantage with this approach is that perhaps we didn't cover as much material as we might have otherwise; maybe we could have finished chapter 5 (Betti numbers, Euler characteristic, map coloring, integral homology, ...) with a more conventional lecture-based approach.