Standard topological terminology

For the following, let n be a non-negative integer; I'll work in n-dimensional Euclidean space, $\mathbf{R}^n$.

Except for "neighborhood", the definitions that Henle gives are equivalent to the standard ones. Rather than the language of nearness, most topology books discuss things in terms of open sets, closed sets, and limit points. Here are the standard definitions, for reference.

Definition.

(a) For any point x of $\mathbf{R}^n$ and any number $\varepsilon > 0$, the *open ball* about x of radius ε is the set

$$B_\varepsilon(x) = \{y \in \mathbf{R}^n : \|x - y\| < \varepsilon\}.$$

(b) A subset A of $\mathbf{R}^n$ is *open* if for every $x \in A$, there is a number $\varepsilon > 0$ so that A contains the ball $B_\varepsilon(x)$.

(c) A *neighborhood* of $x \in \mathbf{R}^n$ is any open set containing x. (Warning: some authors say that a neighborhood of x is any set which contains an open set containing x, so $[0, 1]$ would be a neighborhood of $\frac{1}{2}$, since it contains the open set $(0, 1)$. I don't like this usage, but some people use it.)

(d) A subset A of $\mathbf{R}^n$ is *closed* if its complement $\mathbf{R}^n \setminus A$ is open. One can show that A is closed if and only if A contains all of its near points (this is Henle's definition), or equivalently, if and only if A contains all of its limit points.

(e) Given a set $A \subseteq \mathbf{R}^n$, a point $x \in \mathbf{R}^n$ is a *limit point* of A (also known as a "cluster point" or "accumulation point") if every neighborhood of x contains a point of $A \setminus \{x\}$. Equivalently, x is a limit point of A if for every $\varepsilon > 0$, the open ball $B_\varepsilon(x)$ contains a point of $A \setminus \{x\}$. Equivalently, x is a limit point of A if x is near $A \setminus \{x\}$.

(f) As Henle points out on page 19, the usual definition of continuity involves ε's and δ's.

Compactness is a very important concept in topology. As Henle points out on page 22, when he says "compact", he actually means "sequentially compact". These two notions are actually different (compactness is defined on page 279, if you're interested), but for all of the cases we're going to look at this semester, they end up being the same. So I won't dwell on the "actual" definition of compactness.

As far as connectedness goes, Henle's definition is essentially the standard one.