Math 468 - Final Exam, May 2001

(*Please print*)
Last Name

First name

1. Suppose S is a set in a metric space (X, d). Consider the set

$$\text{S'}:= \Big\{ x \in X; \;\; \exists \text{ sequence } \; s_n \in S \; \text{ such that } \; s_n \neq x, \;\; s_n x \Big\}.$$

(a) Prove that S' is a closed subset of X.
(b) Suppose X is the set of real numbers equipped with the usual metric $d(x, y) = |x - y|$ and S is the set of rational numbers. Find S'.

2. Denote by T the closed subset of ^{3} formed by the edges of a tetrahedron and by C the closed subset formed by the edges of a cube.
(a) Show that both T and C are compact and connected.
(b) Show that T and C are not homeomorphic.

3. Denote by A the annulus $1 \leq x^2 + y^2 \leq 4$ and by C the circle $x^2 + y^2 = 1$.
(a) Show that both A and C are compact and connected.
(b) Show that the set obtained from A by removing a finite collection of points is also connected.
(c) Show that A and C are not homeomorphic.
(d) Denote by D^* the set obtained from the disk $x^2 + y^2 \leq 1$ by removing its center. Prove that A and D^* are not homeomorphic.

4.(a) Give an example of two connected subsets in the plane whose intersection is not connected.
(b) Give an example of a topological space X which contains a subset $\emptyset \subsetneqq S \subsetneqq X$ which is simultaneously closed and open.
(c) Give an example of metric space in which every bounded set is compact.