Takehome Final for Math 468

Due in my office on Wednesday 12/18 at 4:15

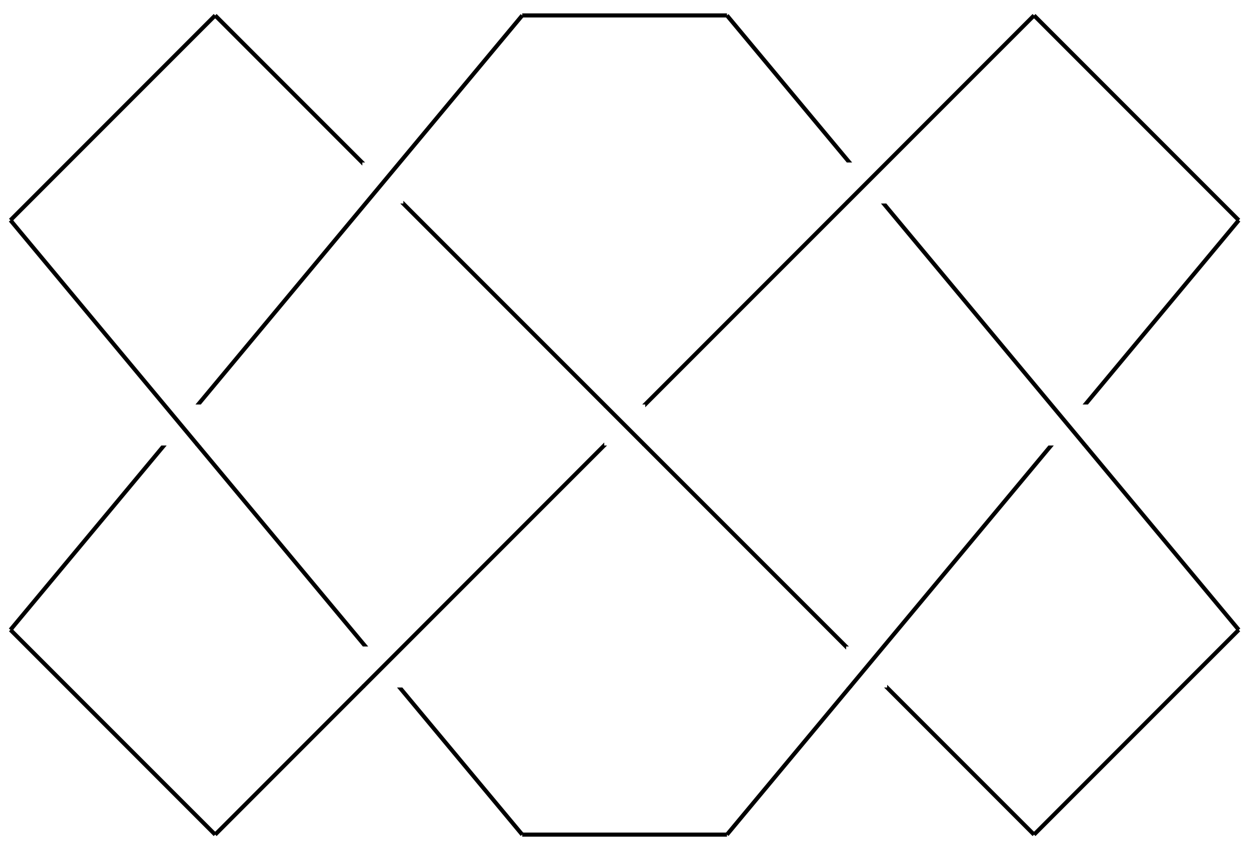
1. Prove part of Lemma 2 from the last handout. Explicitly, show that if f is a homeomorphism from $[a, b]$ to itself of finite order, and if f preserves orientation ($f(a) = a$), then f is the identity.
   Hint: Remember the Intermediate Value Theorem from Math 125 and the result that any open set in the real line is a disjoint union of open intervals.

2. Prove that if a graph G has a vertex of valence 3, then G does not embed in the real line.

3. Let h be any embedding of K_6 in $\mathbf{R}^3$. Show that $\mathrm{Aut}_h(G)$ does not contain both an element of order 5 and an element of order 3.
   Hint: If the is an element of order 5 in $\mathrm{Aut}_h(G)$ the possible configurations of the maximal sublinks are rather restricted.

4. Compute the HOMFLY polynomial of the knot on page 2. Give generators and relations for its group. Show that the group is non–trivial by giving a 3 coloring (or equivalently a surjection onto the dihedral group of order 6). You may wish to make several copies of the knot projection to help describe your answer.

5. There are achiral embeddings of K_8 and K_7 is a subgraph of K_8. But no embedding of K_7 is achiral. Why is not the embedding $K_7 \subset K_8 \to \mathbf{R}^3$ achiral if the embedding $K_8 \to \mathbf{R}^3$ is? As a warm up for the first part of this question, argue that the θ that makes the K_8 embedding achiral can not have precisely one fixed point.



The knot.