For the Oral Candidacy examination, the student is examined in three basic subjects (satisfying the requirements of the
student's intended Track of Specialization). For each subject, the student must master all of the topics listed on the
syllabus. The student is expected to have a through understanding and working knowledge of concepts, theorems,
proofs, examples and counter-examples, and to be able to give a coherent account of this knowledge. Most of the topics
on the syllabus are covered in the basic course sequence, but students are expected to study on their own any topics
which may have been omitted.

The syllabi for the basic subjects are given below, with references for each subject.

ALGEBRA

1. Groups

Subgroups, quotient groups, direct products, the homomorphism theorems. Automorphisms, conjugacy, commutators,
solvability. Theorems of Lagrange, Sylow. EXAMPLES: Cyclic groups, groups of permutations, matrix groups.

2. Rings

Ideals, quotient rings, the homomorphism theorems. Prime and maximal ideals, localization. Local rings, polynomial rings,
Noetherian rings. Principal ideal domains, Euclidean rings, unique factorization domains. EXAMPLES: Rings of
functions, matrix rings.

3. Modules

Theory of modules, projective, finitely generated free modules. The language of categories and functors. Localization of
modules. Structure theory of modules over a principal ideal domain (structure of finitely generated Abelian groups).
Tensor products. Algebras, Wedderburn's theory of simple algebras, Brauer groups. EXAMPLES: Finite dimensional
vector spaces over fields: Basis, dimension, duality, linear transformations and matrices, rank and nullity. Canonical forms
for the matrix of a linear transformation, invariant factors and elementary divisors of a matrix.

4. Fields

Algebraic and transcendental field extensions, degree, transcendence base, algebraic closure, structure of finite fields.
Separability, normal extensions, Galois groups, fundamental theorem of Galois theory. Examples of Galois groups over
the rationals. Cyclotomic extensions, cyclic extensions, theory of equations.

References

T. Hungerford, Algebra, Graduate Texts in Mathematics 73, Springer-Verlag, Berlin-Heidelberg-New York, 1989.

N. Jacobson, Basic algebra I, II, W. H. Freeman and Company, San Francisco, 1985, 1989.

S. Lang, Algebra, Addison-Wesley, Reading, MA, 1984.

REAL VARIABLES

1. Calculus

Calculus of one and several variables, including line integrals, surface integrals, Stokes' theorem, the Implicit and Inverse
Function Theorems, pointwise and uniform convergence of sequences of functions, integration and differentiation of
sequences, the Weierstrass Approximation Theorem, the existence and uniqueness of solutions of ordinary differential
equations.

2. Lebesgue measure and integration on the real line

Measurable sets, Lebesgue measure, measurable functions, the Lebesgue integral and its relation to the Riemann integral,
convergence theorems, functions of bounded variation, absolute continuity and differentiation of integrals.

3. General measure and integration theory

Measure spaces, measurable functions, integration convergence theorems, signed measures, the Radon-Nikodym
Theorem, product measures, Fubini's Theorem, Tonelli's Theorem.

4. Families of functions

Equicontinuous families and the Arzela-Ascoli Theorem, the Stone-Weierstrass Theorem.

5. Banach spaces

-spaces and their conjugates, the Riesz-Fisher Theorem, the Riesz Representation Theorem for bounded linear functionals on , C(X), the Riesz Representation Theorem for C(X), the Hahn-Banach Theorem, the Closed Graph and Open Mapping Theorems, the Principle of Uniform Boundedness, Alaoglu's Theorem, Hilbert spaces, orthogonal systems, Fourier series, Bessel's inequality, Parseval's formula, convolutions, Fourier transform, distributions, Sobolev spaces. (In regard to the last three topics consult Folland's book--see the references--for an indication of what is expected).

References

Apostol, Mathematical Analysis
Riesz-Nagy, Functional Analysis
Royden, Real Analysis
Rudin, Principles of Mathematical Analysis
Rudin, Real and Complex Analysis
Rudin, Functional Analysis
Simmons, Introduction to Topology and Modern Analysis
Wheeden-Zygmund, Measure and Integration
Folland, Real Analysis

COMPLEX VARIABLES

1. Winding, number, integral along curves. Various definitions of a holomorphic function. Connection with harmonic functions. Cauchy Integral Theorems and Cauchy Integral Formula for closed curves in a domain, and for the boundary of a domain, Poisson Formula. The integral of a holomorphic function and its dependence on the path of integration.
Open Map Theorem, Inverse Function Theorem, maximum and minimum principle, Liouville's Theorem. Uniform convergence of holomorphic functions. Normal families of holomorphic functions. Montel and Vitali Theorems, Picard's Theorem. Power series, Laurent series. Residues and classification of isolated singularities, meromorphic functions.
Divisor of a meromorphic function. Residue Theorem, argument principle, Rouché's Theorem, computation of integrals.
Riemann Mapping Theorem, argument principle. Möbius maps. Schwartz Lemma. Theorems of Mittag-Leffler and Weierstrass. Gamma Function, Riemann Zeta Function, Weierstrass p-function.

2. Definition of complex manifolds and examples. Riemann surfaces. The concepts of divisors, line bundles, differential

forms and Chern forms. The Riemann-Roch Theorem.

References

Ahlfors, Complex Analysis. Excellent standard textbook.
Burchkel, An Introduction to Classical Complex Analysis I (best textbook available).
Conway, Functions of One Complex Variable (excellent standard textbook).
Forster, Lectures on Riemann Surfaces (excellent introduction to Riemann surfaces).
Gunning, Lectures on Riemann Surfaces (beautifully written book, in the style of several complex variables).
Knopp, Theory of Functions I, II, and Problem Books (excellent modern textbook written in the spirit of several complex variables).

TOPOLOGY

I. General Topology

1. Separation properties: Hausdorff, regular, completely regular and normal spaces. Urysohn's Lemma and Tietze's Theorem.
2. Construction of topological spaces: Product and quotient spaces, metric spaces, Baire Category Theorem.
3. Covering properties: compact and locally compact spaces. Tychonoff Theorem. Paracompactness and partitions of unity. Some metrization theorem.
4. Miscellany: Path connectedness and connectedness. Topology for mapping spaces, Ascoli's Theorem.

References

Cullen, Introduction to General Topology
Dugundji, Topology
Kelley, General Topology
Munkres, Topology
Steen, Counterexamples in Topology

Algebraic Topology

1. The fundamental group: covering spaces, VanKampen's Theorem and calculation of fundamental groups of surfaces.
2. Homology: Singular homology and cohomology theory. Eilenberg-Steenrod axioms. The cohomology ring.

Homology calculations via CW complexes. Calculation of the cohomology ring of projective spaces.

3.Homotopy: Exact homotopy sequence of a pair. Hurewicz's Theorem.

4.Manifolds. The Poincaré Duality Theorem.

References

J. Vick, Homology Theory (very readable--homology and manifolds only)
G. Whitehead, Homotopy Theory
E. Spanier, Algebraic Topology (complete--a bit condensed)
A. Dold, Lectures on Algebraic Topology (homology and manifolds only)
C. Maunder, Algebraic Topology (good, complete)
R. Greenberg and J. Harper, Lectures on Algebraic Topology (complete and short)

LOGIC

1. Model Theory

Propositional logic, first order predicate logic. Completeness and Compactness Theorems. Löwenheim-Skolem Theorem, saturated models. Omitting types, $\aleph_0-$-categoricity, $\omega-$stability. Quantifier elimination, structure of definable sets. Examples of theories: algebraically closed fields, real closed fields, Presburger arithmetic, dense and discrete linear orderings.

References

Chang and Keisler, Model Theory
Sacks, Saturated Model Theory
Poizat, Cours de Théorie des Modèles

Recursion Theory

Turing machines, Kleene definition of partial recursive functions, primitive recursive functions. Halting set, recursive sets, recursively enumerable sets. Rice's Theorem, Myhill Isomorphism Theorem, index sets. Turing degrees, jumps of sets and degrees. The Hierarchy Theorem, the Recursion Theorem. R.e. degrees, Friedberg-Muchnik Theorem. Gödel's Incompleteness Theorem, Tarski's Undefinability Theorem.

References

R. I. Soare, Recursively Enumerable Sets and Degrees
N. Cutland, Computability

H. Enderton, A Mathematical Introduction to Logic

Set Theory

Axioms of ZFC, Schröder-Bernstein Theorem, ordinals, ordinal arithmetic, proof and definition by recursion. Cardinals,
cardinal arithmetic, regular and inaccessible cardinals. Well-founded sets and the Levy hierarchy. Absoluteness. The
constructible hierarchy. Consistency of GCH and AC.

References

T. Jech, Set Theory
K. Kunen, Set Theory
P. Cohen, Set Theory and the Continuum Hypothesis