Basic Real Analysis, Math 603-4

Fall 98 - Spring 99

A. Alexandrou Himonas

Real Analysis has its roots in the work of Archimedes(287-212 BC) and other ancient Greek mathematicians who developed techniques for finding areas and volumes. In the Seventeenth century these techniques were further developed by Newton and Leibniz into the theory of Calculus. In the Eighteenth and Nineteenth centuries, the power of calculus was applied to the study of many problems of both practical and theoretical interest. For example, Fourier (1768-1830) used calculus (Fourier series) to solve the heat equation. However, most of the concepts underlying Fourier analysis were understood only in the Twentieth century starting with the development of a new integral by Lebesgue (1900-1950). Today Real Analysis is still a growing subject in Mathematics with applications to problems from Social Sciences, Business and Economics to Engineering and the Physical Sciences. Our objective in this course is to present the essentials of modern analysis together with some of its applications in the study of both practical and theoretical problems. Analysis is a live subject, and we shall try to present it as such.

The central theme of the first semester will be the Lebesgue measure and integral, together with the various notions of convergence of a sequence of functions. In addition we will discuss abstract measures and integrals, being generalizations of the Lebesgue measure and integral. We will start with a review of the notions of limit, continuity, compactness, and the Riemann integral in the n-dimensional Euclidean space. And, we will end with differentiation of measures.

In the second semester we will start with an introduction to Banach and Hilbert spaces, and will continue with L^p spaces and Radon measures. In the last part of the semester we will try to present the following topics: An introduction to Fourier analysis, Sobolev spaces and applications to partial differential equations, an introduction to probability and the Brownian motion, and applications to mathematical economics.

Textbooks

1. **G.B. Folland.** Real Analysis, Modern Techniques and Applications, Willey, ISBN 0-471-80958-6

2. **R.L. Wheeden and A. Zygmund.** Measure and Integral, an Introduction to Real Analysis, Marcel Dekker, ISBN 0-8247-6499-4.

References

1. **M. Reed and B. Simon.** Methods of Modern Mathematical Physics, I: Functional Analysis, Academic Press, 2nd edition, ISBN 0-12-585001-8

2. **E. Evans and R.F. Gariepy.** Measure Theory and Fine Properties of Functions, CRC Press, ISBN 0-8493-7157-0.

3. **G.B. Folland.** Introduction to Partial Differential Equations, Princeton University Press, 2nd Edition, ISBN 0-691-04361-2.

4. **F. John.** Partial Differential Equations, Springer-Verlag, 4th Edition, ISBN 0-387-90609-6.

5. **P. Malliavin.** Integration and Probability, Springer-Verlag, ISBN 0-387-94409-5.

6. **D.W. Strook.** Probability Theory, an Analytic View, Cambridge University Press, ISBN 0-521-43123-9.

7. **R. Durrett.** Stochastic Calculus, CRC Press, ISBN 0-8493-8071-5.

8. **N.L. Stokey and R.E. Lucas, Jr.** Recursive Methods in Economic Dynamics, Harvard University Press, ISBN 0-647-75096-9

Syllabus

1. Preliminaries

- **1.** Real Numbers
- **2.** Metric Spaces
- **3.** Compact sets; The Heine-Borel Theorem
- **4.** Continuity
- **5.** Contraction Mapping Theorem
- **6.** The Basic Theorem for Existence and Uniqueness of Solutions to Ordinary Differential Equations
- **7.** The Ascoli-Arzelá Theorem
- **8.** The Weierstrass Approximation Theorem
- **9.** The Baire Category Theorem
- **10.** Review of he Riemann Integral

2. Measures

- **1.** σ-algebras
- **2.** Measures
- **3.** Outer Measures
- **4.** Borel Measures on $\mathbb{R}$
- **5.** Lebesgue Measure on $\mathbb{R}$

3. Integration.

- **1.** Measurable Functions.
- **2.** Integration of Non-negative Functions