Fall 2000 – Spring 2001, Math 603-604: Real Analysis

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Real analysis is the fundation on which many theory are built.

1. Calculus

Calculus of one and several variables, including line integrals, surface integrals, Stokes' theorem, the Implicit and Inverse Function Theorems, pointwise and uniform convergence of sequences of functions, integration and differentiation of sequences, the Weierstrass Approximation Theorem, the existence and uniqueness of solutions of ordinary differential equations.

2. Lebesgue measure and integration on the real line

Measurable sets, Lebesgue measure, measurable functions, the Lebesgue integral and its relation to the Riemann integral, convergence theorems, functions of bounded variation, absolute continuity and differentiation of integrals.

3. General measure and integration theory

Measure spaces, measurable functions, integration convergence theorems, signed measures, the Radon-Nikodym Theorem, product measures, Fubini's Theorem, Tonelli's Theorem.

4. Families of functions

Equicontinuous families and the Arzela-Ascoli Theorem, the Stone-Weierstrass Theorem.

5. Banach spaces

Dual-spaces and their conjugates, the Riesz-Fisher Theorem, the Riesz Representation Theorem for bounded linear functionals on , C(X), the Riesz Representation Theorem for C(X), the Hahn-Banach Theorem, the Closed Graph and Open Mapping Theorems, the Principle of Uniform Boundedness, Alaoglu's Theorem, Hilbert spaces, orthogonal systems, Fourier series, Bessel's inequality, Parseval's formula, convolutions, Fourier transform, distributions, Sobolev spaces. (In regard to the last three topics consult Folland's book–see the references–for an indication of what is expected).

Textbook: Wheeden-Zygmund, Measure and Integral.