Math 605, Fall 2000

Location and Time: Monday, Wednesday, and Friday from 1:55 - 2:45 PM in COMP 227.

Instructor: Jeff Diller. You can reach me at my office 356 CCMB, by phone at 631-7694, or by email at ``diller.1@nd.edu''. My office hours are officially from 3-5 PM Monday and 3-4 PM Wednesday. Feel free to stop by at other times on Monday, Wednesday, and Friday, too, but you might want to call or email first to make sure I'll be in. I'll be a little more jealous of Tuesdays and Thursdays since I like to keep a day or so free for my own research.

Textbook: Function Theory of One Complex Variable by Robert E. Greene and Steven G. Krantz. I have also placed some books on reserve in the math library on the second floor of CCMB:

Complex Analysis by Lars Ahlfors.
Functions of One Complex Variable by John Conway.
Real and Complex Analysis by Walter Rudin.
An introduction to complex function theory by Bruce Palka.
Complex Analysis: the Geometric Viewpoint by Steven G. Krantz.
Theory of Complex Functions by Reinhold Remmert.

No single textbook can be expected to present all points of view on this subject, so if you don't find Greene and Krantz helpful, I'd suggest looking at some of the reserve books, or browsing the math library stacks for books whose numbers begin with QA331. Ahlfors (who died recently) was one of the greatest complex analysts of this century, and his book is a favorite among mathematicians. Conway's book is similar to Ahlfors but heavier on details, a feature many beginning students appreciate. Rudin's book integrates real and complex analysis into a single presentation. Palka's book is the most detailed of all the reserve books. It's also a good source for further problems to practice on. The reserve book by Krantz isn't really a text book at all, but an inspiring and award winning monograph on the use of geometric ideas in complex analysis. I'd suggest having a look at it once we're reach, say, the Riemann mapping theorem. Finally, the book by Remmert gives a history of the material we'll be covering in class.

What is complex analysis? ``Calculus meets complex numbers'' might serve as a starting description of complex analysis, but this doesn't do justice to the potency of the combination. The notion of ``imaginary numbers'', shunned for its apparent absurdity and invoked for its usefulness, has been around since at least the Renaissance. But systematic attempts to take the notion seriously and to integrate it into algebra, analysis, and geometry only really got going in the nineteenth century with the work of Cauchy, Riemann and others. Many facts (e.g. the prime number theorem, the fundamental theorem of algebra) that ostensibly belong to other areas of mathematics are difficult, if not impossible, to state or prove without complex analysis. And many physical theories (e.g. signal processing, quantum mechanics) are most naturally expressed in terms of complex analysis. In the first term of this two semester sequence, I hope to present a large part of the ``classical (i.e. 19th century) theory'' of complex analysis.

What this course will cover: Topics for the first semester are fairly standard. Namely, I hope to cover a largepart of the classical (i.e. nineteenth century) theory of the subject. A more precise list of topics, in roughly the order we'll meet them, is as follows.

Geometry and arithmetic of complex numbers.

Definition and basic properties of complex analytic functions.
Contour integrals and Cauchy's Theorems.
Consequences and applications of Cauchy's Integral formula, including but not limited to Liouville's theorem; the maximum principle; isolated singularities; Calculus of residues;
The general form of Cauchy's theorems.
Conformal Mappings
Normal families and the Riemann mapping theorem.
(The Poincar\'e metric)
(Schwarz-Christoffel transformations)
Harmonic Functions
(Subharmonic functions and the Dirichlet problem)
(Monodromy, Elliptic modular functions, and Picard's Theorems)

Parenthetic topics are things I'd like to cover if time permits. Time is, however, a rather unforgiving taskmaster.

Homework: Homework problems will account for 50% of your grade in this course. I'll assign new problems and pick up your solutions to old ones almost every Friday in class. Note that I don't intend to grade every last problem---I prefer grading a couple of problems well to grading a bunch of them haphazardly. At any rate, I plan to write up solutions to all the problems and to make them available to you.

Exams: There will be a midterm and final exam in this course. They'll be worth 20% and 30% of your grade, respectively. The midterm will take place Monday, October 9 at 7 PM, and the final on Tuesday, December 12 from 8-10 AM in our usual classroom.