Math 605, Fall 2001

Xavier

SYLLABUS

Metric spaces, convergence, continuity, compacteness and connectedness, uniform convergence. The complex plane and the Riemann sphere. Power series and their convergence. Analytic functions, analytic functions as real conformal maps, the real and complex inverse function theorems, the Cauchy-Riemann equations, Mobius transformations. Complex integration, various forms of the Cauchy theorem and the Cauchy integral formula. Cauchy's estimates and applications to Liouville's theorem and the fundamental theorem of algebra. Theorems of Morera and Goursat. The open mapping theorem. Isolated singularities. The Casoratti theorem. The unique continuation principle. Calculation of integrals by the method of residues. Counting zeros and poles, the argument principle. The maximum modulus theorem. Rouche's theorem, Hurwitz's theorem and Schwarz's lemma. Automorphisms of the plane and the disc. Topology on the space of holomorphic functions, proof of Montel's theorem. Proof of the Riemann mapping theorem.

Textbook: Conway's Functions of one complex variable, with eventual use of Rudin's Real and complex Analysis.