Assignment 1
Math 605, Fall '00

Read sections 1.1–1.4 and 2.2 from Greene and Krantz.

Solve the following problems.

1. From Green and Krantz. Pages 22–29: 9, 32, 34, 49 (prove only the $\frac{\partial}{\partial z}$ formula).

2. Since $\mathbf{C}$ is really just $\mathbf{R}^2$ dressed up with a method for multiplying points together, we can think of $\mathbf{C}$ as either a two dimensional real vector space (with e.g. basis $\{(1,0),(0,1)\}$) or a one dimensional complex vector space (with e.g. basis $1+0i$). (If you've never encountered one before, a complex vector space is a vector space in which we're allowed to multiply vectors by *complex* numbers.)

Here we contrast the two corresponding notions of linearity. Recall that a transformation $T : \mathbf{R}^2 \to \mathbf{R}^2$ is *real linear* if

1. $T(z) + T(w) = T(z+w)$;
2. $T(\lambda z) = \lambda T(z)$;

for every $z, w \in \mathbf{R}^2 \cong \mathbf{C}$ and every $\lambda \in \mathbf{R}$. Likewise T is *complex linear* if property 1. holds unchanged, but property 2. is strengthened to hold for every $\lambda \in \mathbf{C}$.

- Show that any real linear transformation can be written in the form $T(z) = Az + B\bar{z}$. Express A and B in terms of the (real, 2x2) matrix for T and vice versa.
- Show that a real linear T is complex linear if and only if $T(z) = Az$ for for all z—i.e. B vanishes. What form must the matrix for T assume in this case?
- Express the determinant of the matrix for T in terms of A and B. What condition on A and B guarantees that T is invertible?

3. (From Ahlfor's book) Find all solutions $z \in \mathbf{C}$ of $\bar{z} = z^n$.