Assignment 11
Math 605, Fall '00

Read sections 7.4–7.7 from Greene and Krantz.

Solve the following problems.

1. From the textbook. Pages 244–254: 1 (do this for harmonic functions which are merely bounded *above*), 18

2. Let $U \subset \mathbf{C}$ be a simply connected domain not equal to $\mathbf{C}$, and let $\varphi : U \to D(0,1)$ be a conformal map. Given a curve $\gamma : [a,b] \to U$, define $L_U(\gamma) = L_P(\varphi \circ \gamma)$.

- Show that
$$L_U(\gamma) = \int_\gamma \frac{|\varphi'(z)|}{1 - |\varphi(z)|^2} \, |dz|$$
Compute the integrand explicitly for $U = \mathbf{H}$.
- Explain why $L_U(\gamma)$ is independent of which conformal map $\varphi : U \to D(0,1)$ we pick.
- Let V be another simply connected domain not equal to $\mathbf{C}$. Let $h : U \to V$ be holomorphic. Show that
$$L_V(h \circ \gamma) \leq L_U(\gamma)$$
with equality if and only if h is a biholomorphism.

3. Let $U \subset \mathbf{C}$ be open. Let $T : \mathbf{R} \to \mathbf{R}$ be a function. Find necessary and sufficient conditions on a C^2 function T such that $T \circ h$ is harmonic whenever $h : U \to \mathbf{C}$ is. Prove that your answer is correct.