Assignment 8
Math 605, Fall '00

Read sections 6.1–6.3 from Greene and Krantz.

Solve the following problems.

1. From the textbook. Pages 147–158: 50, 54, 58.

2. Compute $\displaystyle\int_{|z|=2} e^{e^{1/z}}\,dz$.

3. Suppose that $f : D^*(0,1) \to \mathbf{C}$ is holomorphic. Show that $e^{f(z)}$ cannot have a pole at $z = 0$.

4. One can use the Fundamental Theorem of Algebra and induction to show that a holomorphic polynomial $P(z) = a_0 + \ldots a_k z^k$ of degree k has k complex roots (counted with multiplicity). Give alternative proofs of this fact using

- the Argument Principle;
- Rouche's Theorem.

5. Suppose that $U \subset \mathbf{C}$ is open and for each $n \in \mathbf{N}$, $f_n : U \to \mathbf{C}$ is holomorphic and injective. If f_n converges uniformly locally to $f : U \to \mathbf{C}$ as $n \to \infty$, then show that $f : U \to \mathbf{C}$ is either constant or injective.