Math 605: Complex Analysis I
Fall 2000, MWF 10:40–11:30
Instructor: Jeff Diller

Complex analysis is the study of functions which are complex differentiable—Calculus meets i, so to speak. These functions have applications in fields of mathematics as diverse as geometry, topology, number theory, dynamical systems, and partial differential equations, not to mention many applications outside mathematics. In the first semester of this two semester sequence, we cover the following list of topics which are more or less classical by now. The main theme throughout will be the interplay of the analytic and geometric approaches to the subject.

Topics:

The complex plane: arithmetic and geometry of complex numbers; the Riemann sphere.

Complex differentiable functions: definition and basic properties; Cauchy-Riemann equations; geometric interpretation—conformal mappings.

Examples: linear fractional transformations; exponential and logarithm functions; power series.

Complex integration: definitions and basics; Cauchy's theorem; Cauchy's integral formula for a disk; applications; the maximum principle and more applications; the general form of Cauchy's integral formula.

Harmonic functions: definition and basic properties; Poisson integral formula; applications.

Conformal mapping revisited: normal families and Montel's Theorem; the Riemann mapping theorem; Carathéodory's Theorem; the reflection principle; Schwarz-Christoffel formula; Picard Theorems and the strong version of Montel's Theorem.