amsppt MATH 606

1. Let Ω be a domain in$C$, $a \in \Omega$ and $B_R(a) = z; |z-a| < R \subset \Omega$. Prove the following mean-value property for any $f \in A(\Omega)$:

i) $f(a) = 12\pi \int_0^2 \pi f(a + Re^i\theta) d\theta.$

i i) $f(a) = 1\pi R^2 \quad B_R(a) \quad f(z) d\ x\ d\ y.$

2. Let Ω be a simply connected domain in C and U is a real harmonic function in Ω.

i) Prove that there exists an f which is holomorphic in Ω and U = Ref.

i i) Show that this fails in every region which is not simply connected.

i ii) Show that U^2 cannot be harmonic in Ω unless U is a constant.

3.) Let F be the family of all functions $\{f\} such that \mathbf{f} is holomorphic in the open unit disc$ D = $\{z; \quad |z| <\}$ and f (0) = 1, Re f $>$ 0. Show that F is a normal family.

4.

i) Let $\{fn(z)\}$ be a sequence of entire functions which converges uniformly on compact subsets of C to $f(z)$. Prove that $f(z)$ is entire.

2) Show that $\sum_{n=1}^{\infty} (sinz)^n N^n$ defines an entire function.

5.

i) State and prove the Mittag-Leffler theorem for the whole plane without using Rumge's theorem.

i i) State the Rumge's theorem for a bounded domain Ω and apply the Rumge's theorem to prove the Mittag-Leffler theorem for bounded domain Ω.

6.

a) Let $a \epsilon\ C$ and $\Omega = C$