

**University of Notre Dame**  
**Mathematics Department**  
**MATH 607, Fall 1997**

**Alex Himonas**

Real Analysis has its roots in the work of Archimedes and other ancient Greek mathematicians who developed techniques for finding areas and volumes. In the Seventeenth Century these techniques were further developed by Newton and Leibniz into the theory of Calculus. In the Eighteenth and Nineteenth Centuries, the power of calculus was applied to the study of many problems of both practical and theoretical interest. For example, Fourier (1768-1830) used calculus (Fourier series) to solve the heat equation. However, most of the concepts underlying Fourier analyses were understood in the Twentieth Century starting with the development of a new integral by Lebesgue (1900-1950).

**CONTENT:**

The Lebesgue measure and integral, together with the various notions of convergence of a sequence of functions, will form the central theme of the first semester. In addition, we shall discuss the relation between integration and differentiation. We will start with a review of the notions of the limit, continuity, compactness, and the Riemann integral in the  $n$ -dimensional Euclidian space. And, we will end with abstract measures and integrals being a straightforward generalization of the Lebesgue measure and integral.

We will begin the second semester with an introduction to Banach and Hilbert spaces, and will continue with  $L^p$  spaces and Radon measures. The last part of the semester will cover:

- An introduction to Fourier Analysis;
- Sobolev spaces and applications to partial differential equations;
- An introduction to probability and the Brownian motion;
- Contraction mapping theorem with applications to mathematical economics and the proof of existence and uniqueness of solutions for a differential equation;
- Spectral theory for compact self-adjoint operators on a Hilbert space.

The objective in this course is to present the essentials of modern analysis together with some of its applications in the study of both practical and theoretical problems. Analysis is a live subject, and we shall try to present it as such.

**TEXTBOOKS:**

1. Folland, G. B., *Real Analysis, Modern Techniques and Application*, Willey, ISBN 0-471-80958-6.
2. Sheeden, R. L. and A. Zygmund, *Measure and Integral, An Introduction to Real Analysis*, Marcel Dekker, ISBN 0-8247-6499-4.